Statistics of the largest geomagnetic storms per solar cycle (1844–1993)

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Abstract. A previous application of extreme-value statistics to the first, second and third largest geomagnetic storms per solar cycle for nine solar cycles is extended to fourteen solar cycles (1844–1993). The intensity of a geomagnetic storm is measured by the magnitude of the daily aa index, rather than the half-daily aa index used previously. Values of the conventional aa index (1868–1993), supplemented by the Helsinki Ak index (1844–1880), provide an almost continuous, and largely homogeneous, daily measure of geomagnetic activity over an interval of 150 years. As in the earlier investigation, analytic expressions giving the probabilities of the three greatest storms (extreme values) per solar cycle, as continuous functions of storm magnitude (aa), are obtained by least-squares fitting of the observations to the appropriate theoretical extreme-value probability functions. These expressions are used to obtain the statistical characteristics of the extreme values; namely, the mode, median, mean, standard deviation and relative dispersion. Since the Ak index may not provide an entirely homogeneous extension of the aa index, the statistical analysis is performed separately for twelve solar cycles (1868–1993), as well as nine solar cycles (1868–1967). The results are utilized to determine the expected ranges of the extreme values as a function of the number of solar cycles. For fourteen solar cycles, the expected ranges of the daily aa index for the first, second and third largest geomagnetic storms per solar cycle decrease monotonically in magnitude, contrary to the situation for the half-daily aa index over nine solar cycles. The observed range of the first extreme daily aa index for fourteen solar cycles is 159–352 nT and for twelve solar cycles is 215–352 nT. In a group of 100 solar cycles the expected ranges are expanded to 137–539 and 177–511 nT, which represent increases of 108% and 144% in the respective ranges. Thus there is at least a 99% probability that the daily aa index will satisfy the condition aa < 550 for the largest geomagnetic storm in the next 100 solar cycles. The statistical analysis is used to infer that remarkable conjugate auroral observations on the night of 16 September 1770, which were recorded during the first voyage of Captain Cook to Australia, occurred during an intense geomagnetic storm.

1 Introduction

Siscoe (1976) applied the statistics of extremes to the first, second and third largest geomagnetic storms in nine solar cycles (viz. 11 to 19), as measured by the average half-daily aa index. His study is extended here to fourteen solar cycles (viz. 9 to 22) using the average daily aa index for the interval 1844–1993. The conventional daily aa index (Mayaud, 1980) is available electronically through the National Geophysical Data Center, Boulder, Colorado, for the interval 1868–1993, which extends by almost three solar cycles the time-interval considered by Siscoe. Moreover, the daily aa index has recently been extended backwards in time by two solar cycles (1844–1868), using hourly measurements of magnetic declination made at the Helsinki Magnetic Observatory during the interval 1844–1880 (Nevanlinna and Kataja, 1993). Daily values of the “essentially equivalent” Helsinki magnetic activity index Ak are available electronically through the Finnish Meteorological Institute, Helsinki (Nevanlinna, 1995). Therefore, it is now possible to investigate the statistics of the first, second and third largest geomagnetic storms per solar cycle over a 150-year interval (1844–1993), using essentially homogeneous daily values of the aa index of geomagnetic activity.

However, since the conventional daily aa index is consistently homogeneous for only twelve solar cycles (viz. 11 to 22, as discussed in Sect. 2), the statistics of the largest geomagnetic storms per solar cycle are also
studied for the shorter 126-year interval (1868–1993), as well as the 100-year interval (1868–1967) considered by Siscoe (1976). It is also important to consider all three cases separately because there are 137 missing daily values of the Ak index in the interval 1844–1867, which might just possibly influence the results for fourteen solar cycles. However, the majority of these missing values (83) lie in the interval 19 July 1856–9 October 1856, which is close to sunspot minimum (1856.0).

2 Derivation of the aa and Ak indices

The aa index is derived from hand-scaled magnetograms from two almost antipodal observatories (at invariant magnetic latitudes of approximately ±50°); one in the United Kingdom and the other in Australia (Mayaud, 1980; Menvielle and Berthelier, 1991). The subauroral observatories that have contributed to the derivation of the aa index are Greenwich (1868–1925), Abinger (1926–1956) and Hartland (1957 – present) for the Northern Hemisphere; Melbourne (1868–1919), Toolangi (1920–1978) and Canberra (1979 – present) for the Southern Hemisphere.

For each 3-h interval (00:00–03:00, 03:00–06:00, etc.), K indices (Mayaud, 1980; Menvielle and Berthelier, 1991; Joselyn, 1995) are derived for the two antipodal observatories. The K index is a quasi-logarithmic number between 0 and 9 that is assigned to the end of these specified 3-h intervals. It is derived by measuring the maximum deviation [in nanoteslas (nT)] of the observed field from the expected quiet-time level, for each of the three magnetic-field components (Joselyn, 1995). The largest of the three maxima at each observatory is converted to a K index by using a look-up table appropriate to that observatory. The K indices measured at the two antipodal observatories are then converted back into amplitudes and an individual aa index is the average of the two amplitudes, weighted to allow for the small difference in latitude of the northern and southern observatories, or for the slight changes in the locations of the two antipodal observatories.

Since an individual three-hourly value of the aa index is derived from just two K indices, it provides only an approximate indication of the actual level of planetary geomagnetic activity. However, half-daily, or daily, averages of the aa index give an acceptably accurate indication of geomagnetic activity on a global scale and over a significantly longer time-interval than any other index of geomagnetic activity (Menvielle and Berthelier, 1991). Mayaud (1973) published a unique 100-year (1868–1967) series of three-hourly aa indices, which is based on K indices that were all measured by the author himself to ensure the homogeneity of the time-series. The aa indices have continued to be published subsequently, in order to provide a rapidly available worldwide index of geomagnetic activity that is physically meaningful on a half-daily or daily time-scale.

Nevanlinna and Kataja (1993) have essentially extended the aa index backwards in time by more than two solar cycles (viz. 9 and 10); namely, from 1 July 1844 to 31 December 1867. Their daily index, designated Ak, has been derived from hourly readings of declination (D) made at the Helsinki Magnetic Observatory (60° 10.3’ N, 24° 59.0’ E) during the interval 1844–1880 (Nevanlinna et al., 1993; Nevanlinna and Ketola, 1993). Specifically, three-hourly K indices and daily Ak amplitudes have been computed from declination values using an algorithm developed for the automatic production of K indices in the case of modern digital data (Sucksdorf et al., 1991). To ensure that the Helsinki K indices are as close as possible to the real ones, the percentage occurrence rate of K values in each bin was adjusted to be the same as the corresponding distribution at the present-day Nurmijärvi Observatory, which is only 40 km from Helsinki. This was achieved by varying the Helsinki K = 9 lower limit (K9); reasonably good agreement was found by fixing K9 = 200 nT. Then the three-hourly K indices were converted into daily Ak amplitudes in the range 0–400 nT.

3 Attributes and limitations of the extended aa index

The aa index is used in the statistical study of the largest geomagnetic storms per solar cycle because it spans a much longer time-interval than any other index of geomagnetic activity (Mayaud, 1980; Menvielle and Berthelier, 1991). In addition, the aa index comprises a consistently uniform, homogeneous time-series, at least since 1868, as required by the statistical analysis. For the period of overlap (1868–1880) between the conventional aa index and the Helsinki Ak index, the linear relationship between the monthly means of the two indices is $aa = 0.90 \pm 0.36 + 1.07 \pm 0.02 \cdot Ak$, with a linear correlation coefficient of 0.96 (Nevanlinna and Kataja, 1993). Owing to the local-time character of magnetic disturbances at Helsinki, Nevanlinna and Kataja (1993) have expressed strong reservations about comparing the daily aa and Ak indices. However, if days of missing Ak values are simply ignored, the corresponding linear relationship between the daily values of these indices is $aa = 2.71 \pm 0.19 + 0.93 \pm 0.01 \cdot Ak$, with a linear correlation coefficient of 0.83. This correlation is highly significant for 4415 daily values (there are 150 missing daily values of $Ak$ in the interval 1 January 1868–30 June 1880), according to Student’s t-test (Weatherburn, 1952). However, since the linear relationship between the daily values of aa and Ak yields scaled (“corrected”) values of $Ak$ that are smaller than the original values if $Ak > 38.7$, the actual tabulated (“uncorrected”) values of $Ak$ are used in this study of extremes in geomagnetic activity.

Siscoe (1976) elected to use the half-daily aa index for two reasons: (1) it is referred to universal time rather than “storm time”, which avoids any possible ambiguity relating to the actual definition of storm time; and (2) a time-scale of 12 h is intermediate between the substorm time-scale (~1 h) and the storm time-scale (~24 h). Hence the half-daily aa index filters out substorm variations but retains the storm variation. The daily aa index is used in this paper because half-daily values of...