$^6\text{He} + ^6\text{He} \text{ Clustering of } ^{12}\text{Be in a Microscopic Algebraic Approach}$

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Abstract. The norm kernel of the $A = 12$ system composed of two $^6\text{He}$ clusters, and the $L = 0$ basis functions (in the $SU(3)$ and angular momentum-coupled schemes) are analytically obtained in the Fock-Bargmann space. The norm kernel has a diagonal form in the former basis, but the asymptotic conditions are naturally defined in the latter one. The system is a good illustration for the method of projection of the norm kernel to the basis functions in the presence of $SU(3)$ degeneracy that was proposed by the authors. The coupled-channel problem is considered in the algebraic version of the resonating-group method, with the multiple decay thresholds being properly accounted for. The structure of the ground state of $^{12}\text{Be}$ obtained in the approximation of zero-range nuclear force is compared with the shell-model predictions. In the continuum part of the spectrum, the $S$-matrix is constructed, the asymptotic normalization coefficients are deduced and their energy dependence is analyzed.

1 Introduction

In a number of known papers [1–3] the resonating-group method (RGM) has been applied to studies of collisions between light magic nuclei. The fact that it takes a considerable amount of energy to excite these nuclei simplifies the calculations but leaves beyond the scope of the studies multi-channel features of the continuum spectra of compound systems. Meanwhile, these features appear naturally in the studies of collisions of light nuclei with open $p$-shell, when even at comparatively low energies inelastic exit channels are open.

From a theoretician’s viewpoint, a relatively simple example of collision of light nuclei with open $p$-shell is the scattering of two $^6\text{He}$ nuclei. Admittedly, at
present it is difficult to stage such an experiment, but continuum states of $^{12}$Be populated at the intermediate stage of this scattering are of significant interest. Along with kinematical and dynamical factors, the Pauli exclusion principle is an important ingredient in the formation of these states, and it should be taken into account precisely to understand its role in multi-channel processes. Finally, a theoretical analysis of the co-existence of open and closed channels and its influence to the formation of the continuum spectrum of $^{12}$Be helps in clarifying the significance of the closed channels in the structure of wave functions in the continuum.

Experimental studies of the break-up of $^{12}$Be by Freer et al. [4, 5] and an investigation of excited states of this nucleus in the reaction of two-neutron removal in an exotic $^{14}$Be beam [6] show that there are, in the energy interval between 12 and 25 MeV, states of $^{12}$Be that decay primarily through $^6$He $+ ^6$He and $^8$He $+ ^4$He channels. Based on the experimental data is an assumption that there are states in $^{12}$Be with $^6$He $+ ^6$He cluster structure. This assumption is supported by the calculations in the antisymmetrized molecular dynamics [7, 8], and in a quasi-microscopic coupled-channel model [9] where this decay channel is dominant.

Both decay channels were considered by Descouvemont et al. [10], where cluster states of $^{12}$Be were calculated in a generator-coordinate model. Having analyzed partial widths of the resonance states, the authors pointed out a significant mixing of the cluster configurations.

In this work, we consider two colliding $^6$He nuclei in a microscopic framework – that of the algebraic version of the RGM (AVRGM). The kinematical information is extracted from the norm kernel constructed from the single-particle orbitals [11] which are the kernels of the integral Bargmann transform [12]. Thus the norm kernel (Sect. 2) is defined in the Fock-Bargmann space, and there it can be expanded over the map of the oscillator basis.

Calculations of the norm kernels of several nuclear cluster systems were earlier made by Hecht et al. (ref. [13] and references therein) and Fujiwara et al. [1]. Both groups utilize the Bargmann space technique [14] and the $SU(3)$-scalar property of the norm kernels. Apart from the most tractable, so-called alpha-conjugated systems ($A = 4n$) [15], the to our case most relevant example of $^6$Li $+ ^6$Li was considered in ref. [13], where the norm kernel is tabulated. The projection of the kernel to the basis states required $SU(3)$ Wigner coefficients [16, 17]. If, however, both basis and the norm kernel are known in their explicit analytical form, the projection can be done without any complicated $SU(3)$ recoupling [18]. The case of degeneration in the $SU(3)$ basis needs a special consideration, and a way to resolve the degeneracy is shown in this paper (Sect. 3).

In Sect. 4, we discuss the functions of the angular momentum-coupled (“physical”) basis, which is employed to find the asymptotic behavior of the coefficients of the expansion of the wave function in the $SU(3)$ basis and to take account of the different energies of several decay thresholds, including those with one of the clusters, or both of them, excited. The relationships between the two bases are established there.

Action of the antisymmetrizer can be reproduced by means of an effective potential, properties of which are discussed in Sect. 5. In Sect. 6 it is shown how the matrix elements of the Hamiltonian between the basis functions are