Abstract
In the past decade, there has been substantial progress in the derivation of nuclear forces from chiral effective field theory. Accurate two-nucleon forces have been constructed at next-to-next-to-next-to-leading order (N^3LO) and applied (together with three-nucleon forces at NNLO) to nuclear few- and many-body systems—with a good deal of success. This may suggest that the 80-year old nuclear force problem has finally been cracked. Not so! Some pretty basic issues are still unresolved. In this talk, I focus on the two most pressing ones, namely, the proper renormalization of the two-nucleon potential and subleading many-body forces.

1 Introduction

The problem of a proper derivation of nuclear forces is as old as nuclear physics itself, namely, almost 80 years [1]. The modern view is that, since the nuclear force is a manifestation of strong interactions, any serious derivation has to start from quantum chromodynamics (QCD). However, the well-known problem with QCD is that it is non-perturbative in the low-energy regime characteristic for nuclear physics. For many years this fact was perceived as the great obstacle for a derivation of nuclear forces from QCD—impossible to overcome except by lattice QCD.

The effective field theory (EFT) concept has shown the way out of this dilemma. For the development of an EFT, it is crucial to identify a separation of scales. In the hadron spectrum, a large gap between the masses of the pions and the masses of the vector mesons, like \( \rho(770) \) and \( \omega(782) \), can clearly be identified. Thus, it is natural to assume that the pion mass sets the soft scale, \( Q \sim m_\pi \), and the rho mass the hard scale, \( \Lambda_\chi \sim m_\rho \sim 1 \text{ GeV} \), also known as the chiral-symmetry breaking scale. This is suggestive of considering a low-energy expansion arranged in terms of the soft scale over the hard scale, \( (Q/\Lambda_\chi)^n \), where \( Q \) is generic for an external momentum (nucleon three-momentum or pion four-momentum) or a pion mass. The appropriate degrees of freedom are, obviously, pions and nucleons, and not quark and gluons. To make sure that this EFT is not just another phenomenology, it must have a firm link with QCD. The link is established by having the EFT observe all relevant symmetries of the underlying theory, in particular, the broken chiral symmetry of low-energy QCD [2].

The early applications of chiral perturbation theory (ChPT) focused on systems like \( \pi \pi \) [3] and \( \pi N \) [4], where the Goldstone-boson character of the pion guarantees that the expansion converges. But the past 15 years have also seen great progress in applying ChPT to nuclear forces [5–23]. As a result, nucleon–nucleon (\( N/N \)) potentials of high precision have been constructed, which are based upon ChPT carried to next-to-next-to-next-to-leading order (N^3LO) [19,21,23], and applied in nuclear structure calculations with great success.
However, in spite of this progress, we are not done. Due to the complexity of the nuclear force issue, there are still many subtle and not so subtle open problems. We will not list and discuss all of them, but instead just focus on the two open issues, which we perceive as the most important ones:

- The proper renormalization of chiral nuclear potentials and
- Subleading chiral few-nucleon forces.

2 Renormalization of Chiral Nuclear Forces

2.1 The Chiral $NN$ Potential

In terms of naive dimensional analysis or “Weinberg counting”, the various orders of the irreducible graphs which define the chiral $NN$ potential are given by:

\[ V_{LO} = V^{(0)}_{ct} + V^{(0)}_{1\pi} \]  
\[ V_{NLO} = V_{LO} + V^{(2)}_{ct} + V^{(2)}_{1\pi} \]  
\[ V_{NNLO} = V_{NLO} + V^{(3)}_{1\pi} + V^{(3)}_{2\pi} \]  
\[ V_{N3LO} = V_{NNLO} + V^{(4)}_{ct} + V^{(4)}_{1\pi} + V^{(4)}_{2\pi} + V^{(4)}_{3\pi} \]

where the superscript denotes the order \( \nu \) of the low-momentum expansion. LO stands for leading order, NLO for next-to-leading order, etc. Contact potentials carry the subscript “ct” and pion-exchange potentials can be identified by an obvious subscript.

Multi-pion exchange, which starts at NLO and continues through all higher orders, involves divergent loop integrals that need to be regularized. An elegant way to do this is dimensional regularization which (besides the main nonpolynomial result) typically generates polynomial terms with coefficients that are, in part, infinite or scale dependent [12]. One purpose of the contacts is to absorb all infinities and scale dependencies and make sure that the final result is finite and scale independent. This is the renormalization of the perturbatively calculated $NN$ amplitude (which, by definition, is the “$NN$ potential”). It is very similar to what is done in the ChPT calculations of $\pi\pi$ and $\pi N$ scattering, namely, a renormalization order by order, which is the method of choice for any EFT. Thus, up to this point, the calculation fully meets the standards of an EFT and there are no problems. The perturbative $NN$ amplitude can be used to make model independent predictions for peripheral partial waves [12,13,18].

2.2 Nonperturbative Renormalization of the $NN$ Potential

For calculations of the structure of nuclear few and many-body systems, the lower partial waves are the most important ones. The fact that in $S$ waves we have large scattering lengths and shallow (quasi) bound states indicates that these waves need to be treated nonperturbatively. Following Weinberg’s prescription [5,6], this is accomplished by inserting the potential $V$ into the Lippmann–Schwinger (LS) equation:

\[ T(p', p) = V(p', p) + \int d^3 p'' V(p', p'') \frac{M_N}{p^2 - p''^2 + i\epsilon} T(p'', p), \]  

where $M_N$ denotes the nucleon mass.

In general, the integral in the LS equation is divergent and needs to be regularized. One way to do this is by multiplying $V$ with a regulator function

\[ V(p', p) \rightarrow V(p', p) e^{-(p'/\Lambda)^{2\alpha}} e^{-(p/\Lambda)^{2\alpha}}. \]

Typical choices for the cutoff parameter $\Lambda$ that appears in the regulator are $\Lambda \approx 0.5 \text{ GeV} \ll \Lambda_{\chi} \approx 1 \text{ GeV}$.

It is pretty obvious that results for the $T$-matrix may depend sensitively on the regulator and its cutoff parameter. This is acceptable if one wishes to build models. For example, the meson models of the past [1,24]