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Two Approaches to Holographic Baryons/Nuclei

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Abstract We overview baryons in a string theoretical holographic QCD. In the large N_c limit, the baryon can be viewed in two different ways. The first is a holographic lift of Skyrmion, except that not only pions but also an infinite number of spin 1 mesons are used to construct the solitonic baryon. This approach has been pursued to give an infinitely predictive model of meson–baryon dynamics. After a brief review, we comment on the alternative picture where the baryon is viewed as wrapped D-branes, which leads to a quantum mechanical description involving matrices. The two approaches give surprisingly similar answers for some quantities, such as hadronic size of baryons and repulsive cores, even though they represent two very different approximations.

1 Holographic Chiral Dynamics of Mesons and Solitonic Baryons

A holographic description [1–3] of large N_c gauge theories [4] can appear arbitrary, mysterious, and puzzling. However, holography can be characterized at least by three principles that constrain and guide us. The first is that holography replaces a strongly correlated $D = d$ dimensional theory by a weakly coupled $D > d$ dimensional theory. The latter, mathematical spacetime is often called *the bulk*, in comparison to the true spacetime, called *the boundary*, on which the original gauge theory lives. The second is that only gauge singlet degrees of freedom survive in the descriptions, so for QCD one works directly with glueballs, mesons, and baryons. In this sense, a holographic QCD shares a common feature with the Chiral Perturbation theory. The third is that a global symmetry of the original theory is elevated to a gauge symmetry in the bulk. The last also shows why one always ends up with some version of gravitational theory in the bulk; we are typically interested in Lorentz invariant field theories, whose Lorentz symmetry gets elevated to diffeomorphism invariance in the bulk. Similarly, a flavor symmetry in QCD induces a flavor gauge theory in the bulk.

These features are beautifully realized in the D4-D8 model of holographic QCD, pioneered for pure Yang–Mills theory by Witten [5] and later extended to QCD with massless quarks by Sakai and Sugimoto [6]. This D4-D8 model has an unique advantage over other proposed holographic models that at lowest of energy scale we are assured that the theory being studied is indeed QCD, albeit with massless quarks only. Since the theory deviates from QCD at higher energy scale, utility of the model becomes questionable beyond certain scale, typically 1 GeV. However, the model has produced numerous predictions that seem to agree with data remarkably well beyond this naive cut-off.

In the first part of this talk, I would like to outline the model with some emphasis on meson–baryon dynamics. Let us skip the glueball story [7–9] and proceed to mesons. For four-dimensional QCD with N_f massless

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flavors, D4-D8 model tells us to solve the following five-dimensional $U(N_f)$ gauge theory at tree-level [6],

$$-\frac{1}{4}\int dx^4 dw \;\; \frac{1}{e(w)^2}\,\mathrm{tr}\mathcal{F}^2 + \frac{N_c}{24\pi^2}\int_{4+1} \omega_5(\mathcal{A}) \tag{1}$$

with the Chern–Simons 5-form, $\omega_5(\mathcal{A})$, and

$$\frac{1}{e(w)^2} = \frac{\lambda N_c}{108\pi^3}\,u(w)\,M_{KK}\;, \quad \frac{2}{3}\,wM_{KK} = \pm\int_1^u dy/\sqrt{y^3-1}. \tag{2}$$

As is implicit here, w coordinate spans a finite parity-even interval.

Incorporation of baryons to the model proceeds by identifying the latter as the topological soliton of this flavor gauge theory, characterized by the first Pontryagin number,

$$p_1(\mathcal{F}) \equiv \frac{1}{8\pi^2}\int_{x^{1,2,3},w} \mathrm{tr}\,\mathcal{F}\wedge\mathcal{F} = 1 \tag{3}$$

This soliton can be understood as a holographic uplift of Skyrmions, except that it is now coherent state of not only pions but also of an infinite tower of spin 1 mesons, all of which are embedded into the single flavor gauge field $\mathcal{A}$.

For simplicity, let us consider $N_f = 2$. Quantizing the soliton to produce spin 1/2 baryons, and representing them via a local field $\mathcal{B}$, baryon dynamics can be added unambiguously to the meson sector as [10,11]

$$+\int d^4x\,dw\left[-i\bar{\mathcal{B}}\gamma^m D_m\mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{2\pi^2\rho^2_{baryon}}{3e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}\mathcal{B}\right], \tag{4}$$

where

$$m_{\mathcal{B}}(w) = \frac{4\pi^2}{e(w)^2}, \qquad \rho^2_{baryon} \simeq \frac{(2\cdot 3^7\cdot\pi^2/5)^{1/2}}{M^2_{KK}\lambda}. \tag{5}$$

Analog of this for higher spin baryons has been studied in a similar vein [12]. The coupling between mesons and baryons occurs via two interaction terms. The first is embedded into the covariant derivative,

$$D_m \equiv \partial_m - i(N_c\mathcal{A}_m^{U(1)} + A_m), \tag{6}$$

for which the flavor gauge field $\mathcal{A}_m$ of (1) is decomposed as $\mathcal{A}_m^{U(1)} + A_m$ with traceless 2×2 A_m. The second is through a direct coupling to the $SU(2)$ part field strength, F.

One natural question here is why one is allowed to introduce a field $\mathcal{B}$, usually suitable for point-like objects, in place of a soliton. Solitons are usually fluffy and big, which would not be amenable to treatment as point-like objects. The answer is found in the parameter ρ_{baryon} which can be thought of as hadronic size of the soliton in question. This size is computed by inserting the unit Pontryagin number configurations and minimizing the energy functional of (1). In the holographic limit of $N_c \gg 1$, $\lambda \gg 1$, note that

$$\rho_{baryon} \ll 1/M_{KK}, \tag{7}$$

so, strangely enough, the soliton is actually far smaller than the fundamental scale of the meson sector $1/M_{KK}$ [10,13]. For instance, as we see in next section, masses of spin 1 mesons are found as $m_n^2 \sim \epsilon_n M_{KK}^2$ with eigenvalues ϵ_n, smallest of which is ~ 0.6. One way to understand the small size is that mesons with large $\epsilon_n \gg 1$ also enter the construction of the soliton. On the other hand, we are primarily interested in interaction of baryons with low-lying mesons, meaning those of masses of at most a few GeV. For this, we do not lose anything by treating the quantized baryon as point-like objects.

This does not mean [14] that we can forget about the long range flavor gauge fields attached to the soliton, of which there are two kinds: self-dual $SU(2)$ magnetic fields and $U(2)$ electric fields. In fact, the Lagrangian (4) is designed to faithfully reproduce these via flavor gauge field equation of motion [10], and can be trusted when the baryon is small, as guaranteed by $\rho_{baryon}M_{KK} \ll 1$, and stays near $w = 0$, as guaranteed by the shape of $m(w)$. Finally, an important caveat in using this combined theory of mesons and baryons comes from the fact that the baryon field $\mathcal{B}$ came from coherent states of the meson theory, so in using the above action, we must discard such topologically nontrivial sector with $p_1(\mathcal{F}) \neq 0$ to avoid double counting. For simple perturbative processes involving mesons and baryons, of course, this is hardly an issue.