Semileptonic Decays of $\Xi_b$ and $\Sigma_b$ Baryons via the Three-Body Variational Approach

Abstract We consider a nonrelativistic three-body system with a Cornell interaction in hyperspherical formalism. In order to obtain the solution of the system, we use the improved variational method. Next, we investigate the Isgur–Wise function and thereby report the semileptonic decay width of bottom baryons $\Xi_b$ and $\Sigma_b$. We report the branching ratios and partial decay widths as well and make a comparison with present data.

1 Introduction

The heavy quark symmetry (HQS) provides us with valuable information on spectroscopy and weak decays of hadrons containing a heavy quark. The semileptonic decays of bottom to charmed baryons yield significant source of knowledge on the internal structure of hadrons containing a single heavy quark. At this condition, the semileptonic decays of baryons possess a single form factor known as the Isgur–Wise function (IWF) [1] that measures the overlap of hadronic wave functions. The IWF is obtained in several parameterization form in different models such as the MIT bag model [2], QCD sum rule [3], relativistic three quark model [4], Skyrme model [5], simple quark model [6], etc. This difference in parameterization is due to the unknown kinematic dependence of the IWF [7]. The observations of more known bottom baryons i.e. $\Lambda_b$ and their orbitally excited states enrich our knowledge of the bottom baryon family and provide elegant clues for future studies in the field. At the moment, the theory of heavy baryons decay, despite its strengths and successes, is not quite solid and does need further improvement. The results of Bethe–Salpeter equation for semileptonic decay of Lambda baryon should be mentioned as a reliable approach in this annals [8]. Heavy baryons in the Skyrme model with IWF for the semileptonic decay of heavy baryons are discussed in Ref. [9]. Recently, some authors considered the heavy baryon decays within the framework of chiral perturbation theory [10], quark pair creation model [11], covariant confined quark model [12], and also the IWF approach [13,14].

Here, we intend to work with the IWF and therefore our first square is finding the solutions of the corresponding three-boy Schrödinger equation. Hence, in the next section, we obtain the baryonic wave function applying variational method. In Sect. 3, we investigate semileptonic decay width of $\Xi_b$ and $\Sigma_b$ baryons with IWF. Section 4 includes conclusions and discussion on the manuscript.
2 Baryonic Wave Function

We consider baryons as bound states of three quarks. The configuration of three Ω particles are described by two Jacobi vectors, ρ and λ as [15]

\[ \rho = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \quad \lambda = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) \]  

(1)

Instead of the variables ρ and λ one can use the hyperspherical coordinates which include the angles Ω = (θρ, φρ) and Ωλ = (θλ, φλ). By introducing the hyperspherical coordinates, with the hyperradius x and the hyperangle ζ as

\[ x = \sqrt{\rho^2 + \lambda^2}, \quad \zeta = \tan^{-1}\left(\frac{\rho}{\lambda}\right) \]

(2)

respectively, the Hamiltonian of the three-body system is expressed as

\[ H = \frac{p_\rho^2}{2m_\rho} + \frac{p_\lambda^2}{2m_\lambda} + V(x) \]

(3)

where the kinetic energy operator takes the form (\(\hbar = c = 1\))

\[ \left(-\frac{\Delta_\rho}{2m_\rho} + \frac{\Delta_\lambda}{2m_\lambda}\right) = -\frac{1}{2\mu} \left(\frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} - \frac{L^2(\Omega_\rho, \Omega_\lambda, \zeta)}{x^2}\right) \]

(4)

in terms of the angles of hyperspherical coordinates Ω = (θρ, φρ) and Ωλ = (θλ, φλ). The eigenfunction of \(L^2\) are hyperspherical harmonics and [15]

\[ L^2(\Omega_\rho, \Omega_\lambda, \zeta)Y_{\gamma.l_\rho,l_\lambda}(\Omega_\rho, \Omega_\lambda, \zeta) = \gamma(\gamma + 4)Y_{\gamma.l_\rho,l_\lambda}(\Omega_\rho, \Omega_\lambda, \zeta) \]

(5)

Here, the grand-angular momentum \(\gamma = 2n + l_\rho + l_\lambda\) with \(n = 0, 1, \ldots\) and \(l_\rho, l_\lambda\) are the angular momenta associated with the ρ and λ variables. The terms \(m_\rho, m_\lambda, \mu\) are defined in terms of the constituent quark masses as

\[ m_\rho = \frac{2m_1m_2}{m_1 + m_2}, \quad m_\lambda = \frac{m_1m_2m_3}{m_1 + m_2 + m_3}, \quad \mu = \frac{m_\rho m_\lambda}{m_\rho + m_\lambda} \]

(6)

Choosing the Cornell potential [16]

\[ V(x) = \alpha x + \frac{\beta}{x} + V_0 \]

(7)

the corresponding hyperradial Schrödinger equation takes the form

\[ \frac{d^2\psi_{n,\gamma}(x)}{dx^2} + \left[ -\frac{\gamma(\gamma + 4) + 15}{4} - \frac{2\mu\beta}{x} - 2\mu ax + 2\mu(E_{n,\gamma} - V_0) \right] \psi_{n,\gamma}(x) = 0 \]

(8)

where \(E_{n,\gamma}, \psi_{n,\gamma}(x)\), γ are the energy of the system, the hyperradial wave function and the grand angular momentum, respectively. To solve Eq. (8), we use the variational technique [17–19] which is one of the most popular approaches to deal with quantum mechanical few-body problems. By this method, one can get a virtually exact solution with an appropriately chosen function space [19]. We consider the trial hyperradial wave function as

\[ \psi_{n,\gamma}(a, x) = N(ax)^{\frac{3}{2} + \gamma} e^{-ax} L_n^{2\gamma + 4}(ax) \]

(9)

where a is the variational parameter, N denotes the normalization constant and \(L_n^{2\gamma + 4}(ax)\) represents the Laguerre polynomial. Choosing of linear confinement term as parent leads to Airy functions. It is noted that the hyperradial equation with the hypercentral Cornell potential cannot be solved analytically unless the linear