Multiresolution Visualization of Higher Order Adaptive Finite Element Simulations

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Received September 25, 2001; revised March 31, 2003
Published online: May 26, 2003
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Abstract

We propose an appropriate and efficient multiresolution visualization method for piecewise higher order polynomial data on locally refined computational grids. Given some suitable error indicators, we efficiently extract a continuous $h$-$p$-adaptive projection with respect to any prescribed threshold value for the visual error. This projection can then be processed by various local rendering methods, e.g. color coding of data or isosurface extraction. Especially for color coding purposes modern texture capabilities are used to directly render higher polynomial data by superposition of polynomial basis function textures and final color look-up tables. Numerical experiments from CFD clearly demonstrate the applicability and efficiency of our approach.

AMS Subject Classifications: 76M27, 65N30, 65S05.

Keywords: multiresolution visualization, higher order, adaptive methods, error indicator, finite element.

Introduction

Today’s numerical methods for the simulation of continuum mechanical phenomena continuously advance in their performance and algorithmic complexity. Finite element methods provide a widely used tool for the solution of problems with an underlying variational structure. Modern numerical analysis and implementations for finite elements provide more and more methods for the efficient solution of even large-scale applications. Efficiency can be increased by using local mesh adaption, higher order elements, where applicable, and by fast solvers. Adaptive procedures for the numerical solution of partial differential equations started in the late 70s and are now standard tools in science and engineering. Adaptive finite element methods are a powerful approach for handling multi-scale phenomena and making realistic computations feasible, especially in 3D.

The task of visualizing interesting solution features of 2D and 3D data sets, as they result from such adaptive simulations, is thus increasingly demanding, as datasets are usually very large and come along with complex data structures.
describing local function spaces beyond the simplest linear case. Nevertheless interactivity in post-processing is indispensable for effective and efficient post processing and analysis of given data. Multiresolutional techniques have proved to be very powerful tools in that respect. If numerical data is already characterized by its hierarchical structure a corresponding visualization method is required to make use of this structure for post processing purposes too. In [30, 31] a general concept for the visualization of multi-linear finite element data has been presented. Grid cells are supposed to be tensor products of simplices, recursively refined via arbitrary refinement. In [33] this approach has been generalized conceptually to arbitrary nested function spaces. The essential ingredient of the approach is a suitable saturation condition for error indicators corresponding to the available degrees of freedom. This saturation condition turns out to be fairly natural and can be ensured by a preprocessing of data. Basically, the method extracts an adaptive projection of the finite element function in a depth first traversal of the grid hierarchy. This projection can then be forwarded to any local rendering tool on grid cells.

Here, we will focus on the increasingly important class of piecewise higher order polynomial data on locally refined computational grids and derive an efficient multiresolutional algorithm to extract an $h$–$p$–adaptive projection on suitable grid levels and of suitable local polynomial degree (As usual $h$ indicates the grid size function and $p$ the local polynomial degree). Differing from an $h$–$p$ finite element simulation [1, 29, 38] – where the local grid size and polynomial degree has to be chosen freely and efficiently – we suppose already to start with simulation data given on some adaptive grid and characterized by locally different polynomial degrees. Thus, we solely consider the data post processing problem and implicitly assume that the local grid size and polynomial degree are delivered from the simulation in an appropriate problem dependent way – usually representing smooth functions on coarse grids with high polynomial degree and singularities on highly refined grids with small polynomial degree. A user prescribed error threshold controls the tolerance between the actually visualized data and the true simulation result. Given a threshold in the post processing we first recursively traverse the grid hierarchy corresponding to the adaptive fine grid and, where necessary, select the polynomial degree locally on the finest cells. Obviously, ensuring global continuity is the crucial quality criterion.

To ensure real time performance of the visualization method the strategy to solve the continuity problem at element faces has to be very efficient. Thus, we choose in Section 2.2 a Lagrangian basis approach because of its algorithmic simplicity. One especially observes that on simplices of any dimension ($d = 1, 2, 3$) we always have the same type of local function spaces to cope with and the restriction on sub simplices is performed via straight forward trace operators. Let us emphasize that in contrast for $h$–$p$ simulation purposes a true hierarchical $h$–$p$ basis is usually the preferable choice [1].

To support such a multiresolutional strategy local error indicators are not only given with respect to the grid cells or grid nodes but also for the different possible polynomial degrees.