Dynamic Data Driven Simulations in Stochastic Environments

C. Douglas, Lexington, Y. Efendiev, R. Ewing, College Station, V. Ginting, Fort Collins, and R. Lazarov, College Station

Received September 13, 2005; revised March 20, 2006
Published online: June 6, 2006
© Springer-Verlag 2006

Abstract

To improve the predictions in dynamic data driven simulations (DDDAS) for subsurface problems, we propose the permeability update based on observed measurements. Based on measurement errors and a priori information about the permeability field, such as covariance of permeability field and its values at the measurement locations, the permeability field is sampled. This sampling problem is highly nonlinear and Markov chain Monte Carlo (MCMC) method is used. We show that using the sampled realizations of the permeability field, the predictions can be significantly improved and the uncertainties can be assessed for this highly nonlinear problem.

AMS Subject Classifications: 65N99.

Keywords: MCMC, porous media flow, uncertainty, permeability, DDDAS.

1. Introduction

Dynamic data driven simulations (DDDAS) are important for many practical applications. Consider an extreme example of a disaster scenario in which a major waste spill occurs in a subsurface near a clean water aquifer. Sensors can now be used to measure where the contamination is, where the contaminant is going to go, and to monitor the environmental impact of the spill.

One of the objectives of dynamic data driven simulations is to incorporate the sensor data into real-time simulations that run continuously. Unlike traditional approaches, in which a static input data set is used as initial conditions only, our approach assimilates many sets of data and corrects computed errors above a given level (which can change during the course of the simulation) as part of the computational process. Many important issues are involved in DDDAS for this application and some of them are described in [3].

Subsurface formations typically exhibit heterogeneities over a wide range of length scales whereas the sensors are usually located at sparse locations and sparse data from these discrete points in a domain is broad-casted. Because the sensor data usually contains noise, it can be imposed either as a hard or a soft constraint. Previously, to incorporate the sensor data into the simulations, we introduced a multiscale interpolation operator. This is done in the context of general nonlinear
parabolic operators that include many subsurface processes. The main idea of this interpolation is that we do not alter the heterogeneities of the random field that drives the contaminant. Instead, based on the sensor data, we rescale the solution in a manner that it preserves the heterogeneities. This interpolation technique fits nicely with a new multiscale framework for solving nonlinear partial differential equations.

The interpolation technique is only a temporary remedy because it does not correct the error sources that occur in the initial data, as well as in the permeability field. Previously, we addressed the initial data correction. However, one of the main sources of the errors is the permeability field. Indeed, permeability fields are typically given by their covariance matrix and some local measurements. In this setting, one can have large uncertainties, and, consequently, it is important to reduce the uncertainties by incorporating the additional data, such as the available information about the contaminant.

In this paper, our goal is to study the permeability correction. The permeability represents the properties of the porous media. Its correction is substantially different from the correction of the initial data. In particular, the problem of permeability correction is strongly nonlinear and stochastic.

In this paper, we also assume that the covariance of the permeability field and some of its values at the measurement points are known. Using Karhunen-Loève expansion, the permeability is expanded based on the covariance matrix. This allows some dimension reduction, which can be further reduced by incorporating the permeability values at sensor locations. Furthermore, based on measurement errors and a priori information about the permeability field, we consider the sampling of the permeability field. This sampling problem is highly nonlinear and the posterior distribution is multimodal. We use the Markov Chain Monte Carlo (MCMC) method to sample from this multimodal distribution. We show that using the sampled realizations of the permeability field, the predictions can be significantly improved. Moreover, the proposed technique allows assessment of the uncertainties for this highly nonlinear problem. The proposed approach is general and can be applied to more complicated porous media problems.

The paper is organized as follows. In the next section, we describe the methodology. Section 3 is devoted to numerical results. Conclusions are presented in Sect. 4.

2. Methodology

We study the problem of contaminant transport in heterogeneous porous media. The model equations are

$$\frac{\partial C}{\partial t} + v \cdot \nabla C - \nabla \cdot (D \nabla C) = 0 \quad \text{in} \quad \Omega, \quad (1)$$

where by Darcy's Law, $v = -k \nabla p$, in which the pressure $p$ satisfies

$$-\nabla \cdot (k \nabla p) = 0. \quad (2)$$