Systematic Design of Systolic Correlators with Application to Parallel Blackman–Tukey Spectral Estimation

N. A. S. Alwan, Baghdad

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Abstract

Different word-level systolic algorithms exhibiting different properties are derived systematically for the problem of correlation computation by applying certain transformations on the dependence graph which represents the algorithm. The algorithm with properties which best suit the application of Blackman–Tukey spectral estimation is selected and the complete estimator comprising autocorrelation, windowing and DFT computation is implemented systolically. The overall computation time of the parallel system is $6LT_x + 3LT_+$ as compared to $(3L/2 \log_2 L + 2L)T_x + (3L \log_2 L)T_+$ for the serial system with fast autocorrelation and FFT where $L$ is the number of input data samples and $T_x$ and $T_+$ are the times needed for one multiplication and one addition respectively. This indicates that the parallel system has a speed advantage over the serial system for $L \geq 16$ and this speed advantage increases with $L$. For a typical 1024-sample segment, the parallel system is almost three times faster.

The system is also modified to suit applications where a number of spectrum measurements are required consecutively as is the case with iterative spectral estimators. For the modified system, the system clock period can be made as small as $2T_x + T_+$ compared to $[(3/2) \log_2 L + 2]T_x + (3 \log_2 L)T_+$ for the serial system so that the parallel system can operate at a higher rate for any value of $L$.

Key Words: Blackman–Tukey spectral estimation, systematic design of systolic arrays, systolic correlators, systolic DFT.

1. Introduction

The need for spectrum analysis arises in many fields of applications such as the study of communication engineering signals, of event-related or stimulated responses of the human electroencephalogram (EEG) in the diagnosis of brain diseases, of other biological signals and of meteorological data and the measurement of noise spectra for the design of optimal linear filters.

The available spectrum estimation techniques may be categorized as nonparametric and parametric. The nonparametric methods include the periodogram, the Bartlett and Welch modified periodogram and the Blackman–Tukey methods [2, 3, 6]. All these methods have the advantage of possible implementation using the Fast Fourier transform (FFT) but with the disadvantage in the case of short data lengths of limited frequency resolution. Parametric methods, on the other hand, can provide high resolution in addition to being computationally efficient. The most common parametric approach is to derive the spectrum from the parameters of an autoregressive model of the signal [13]. Moving average and autoregressive
moving average models have also been employed [13–15]. The disadvantage of these methods, however, is that considerable time and effort may be necessary to form a sufficiently accurate model of the process from which to estimate the spectrum.

The problem under investigation in this paper is the systolic implementation of a nonparametric spectrum estimator, namely the Blackman–Tukey (BT) estimator. This estimator is characterized by having the largest quality factor among the nonparametric methods where the estimator quality factor is determined as the ratio of the square of the mean of the power spectral density to its variance [2, 5].

The Blackman–Tukey procedure is
1. to calculate the autocorrelation function of the data,
2. to apply a suitable window function to the data and
3. to compute the discrete Fourier transform (DFT) of the resulting data to obtain the power density spectrum.

The autocorrelation function is windowed to taper it towards its extremes because at larger lags fewer data enter the computation so that these estimates are less accurate. Tapering has the effect of attaching less weight to these estimates. When the number of data points, L, of the window is much larger than the lag value, the BT estimate will be asymptotically unbiased, and when L is much larger than the number of samples of the autocorrelation function, the estimate is consistent [2].

Systolic implementation of the BT estimator involves the systolic design of the autocorrelation array and the DFT array. The DFT computation is a matrix-by-vector multiplication which can be carried out systolicly either by inputting the matrix row-wise into the processing elements (PE’s) or cells of the systolic array or by inputting the matrix column-wise [1]. Systolic algorithms exhibiting different properties can result from the systematic design of the correlator array. The choice between these algorithms and between row-wise and column-wise DFT computation is dictated by the requirements of local communication between cells and high speed and/or throughput [12] of the overall spectral estimator.

One methodology of systematic design of systolic arrays, as opposed to ad hoc design, is based on the representation of algorithms by means of dependence graphs. One class of such graphs corresponds to the systolic algorithms, in that the projection of a graph from this class delivers a systolic parallel algorithm and in particular such constituents of the algorithm as the respective systolic array and the required input/output operations. The relation between different systolic algorithms for one and the same problem is evident: applying certain transformations on a graph of this class results in other graphs of the same class which, when projected, give rise to other systolic algorithms which exhibit different properties [1].

In Section 2, various systolic correlator algorithms are derived systematically and the one that best suits the application under consideration is chosen on the grounds that it achieves the optimum local inter-connection pattern and the