2D Airflow over a Double Bell-Shaped Mountain

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With 10 Figures

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Summary

The airflow over an idealized orography with two mountain peaks and a valley between is investigated using a non-linear numerical model. The flow is assumed to be two-dimensional and nonrotational. Surface friction is neglected. This setup is a first step in studying the modifications a finely structured “real” topography introduces to the well-studied flow over one isolated obstacle. The sensitivity of the flow behavior to the valley width is examined for the case of specified mountain volume as well as constant non-dimensional mountain height. Flow patterns for linear, weakly nonlinear, wave breaking and upstream blocking cases are examined. Whereas the nondimensional mountain height is still the main measure of the nonlinearity of the flow, the differing steepness of upslope and downslope caused by the separating valley, strengthens nonlinear effects. It also modifies wave breaking and upstream blocking. For wide enough valleys wave breaking regions can form above both peaks.

1. Introduction

During the last decades airflow over mountains has been intensively investigated using several different methods. The first theoretical studies made by Queney (1948) and Lyra (1940) yielded analytical expressions for the vertical displacement of streamlines of 2D linearized flow over idealized topography in a homogeneously stratified atmosphere. Long’s (1953) analytical model was the first one to allow nonlinear interactions, which are important for large amplification of gravity waves. His simplified governing equations for a 2D incompressible, stably stratified, and steady fluid – in which the upstream dynamic pressure and density gradient are assumed to be constant – were used as a fundamental basis for further analytical studies: Miles and Huppert (1969) found that for a certain critical value of the nondimensional mountain height $H_m = h_mN/U$ (with $h_m$ the crest height, $N$ and $U$ the buoyancy frequency and wind speed of the basic state) the isentropes above the mountain become vertical. This value depends on the mountain shape, and is e.g., 0.85 for a witch of Agnesi profile. Ten years later Lilly and Klemp (1979) compared the solutions of Long’s equation for symmetric and asymmetric mountain profiles. They showed that a significant enhancement of wave amplitude and drag can be observed for obstacles with gentle windward and steep leeward slopes.

Numerical simulations found wave breaking to be an important mechanism for the evolution of downslope windstorms and the transition to a high-drag state. Smith (1985) and Durran (1986) explain the evolution of this new flow regime by comparing it with hydraulic theory, which predicts a transition of shallow-water flow from sub- to supercritical at a critical Froude number $Fr = 1$. Numerical simulations of Bacmeister and Pierrehumbert (1988) and Durran and Klemp (1987) supported this explanation.

Pierrehumbert and Wyman (1985) investigated upstream effects of mountain barriers. Numerical simulations of nonrotating flow over a Gaussian-
shaped mountain profile revealed that the upstream flow is decelerated to rest for $H_m \geq 1.5$. Further, the depth of the stagnant layer $d$ increases as the nondimensional mountain height is made larger. This process, called “orographic adjustment”, keeps the nondimensional depth of the unblocked flow below the crest, $H_m - dN/\nu$, nearly constant at 1.5.

Stein (1992) presented a regime diagram for 2D flow over a bell-shaped ridge based on numerical simulations. The nondimensional drag as a function of nondimensional mountain height $H_m$ was found to be a good predictor for the separation of the flow into three regimes. Whereas in the quasi-linear regime ($H_m \ll 1$) the dimensional drag is similar to its linear value, wave breaking ($H_m \sim 1$) causes a drastic increase of the drag by a factor up to three. A third regime ($H_m \sim 2$) occurs by a further increase of $H_m$; nondimensional drag decreases again due to the onset of blocking of the upstream low-level flow caused by the high mountain.

All these previous 2D flow studies were concerned with a single ridge. However, real topography is much more complex: A single ridge is often divided into several peaks due to valleys between. A multi-structural orography generates a different kind of gravity wave spectrum compared to a single-bell shaped one and therefore modifies the flow-regime diagram. The simplest extension is the flow over twin peaks. Examples are the Inn valley, which separates the northern Alpine range (Nordkette) from its main ridge (meso-gamma scale) and on a much larger scale – the Great Basin, which separates the Sierra Nevada and the Rocky Mountains. For hydrostatic conditions (relatively broad mountains) Vosper’s (1996) numerical simulations with a linearized steady-state model showed good agreement with the analytically derived linear drag (Grisogono et al., 1993) as long as the mountain waves are assumed to be hydrostatic (i.e., $U/(Na) \ll 1$, where $a$ is the mountain half-width). However, if non-hydrostatic effects become important, i.e., $U/(Na) \sim 1$, the dependence of drag on the valley width is complicated due to the superposition of non-hydrostatic trapped lee waves generated by both of the two peaks.

For obstacles taller than mere hills nonlinear phenomena are not only essential for the details but also for the overall structure of the resulting flow. The second step from the extensively investigated isolated obstacle to the double obstacle has not been made yet. The present study will try to fill this gap by undertaking idealized 2D numerical simulations of flow over twin peaks for the whole spectrum of possible flow regimes. Section 2 shows the analytical representation of the investigated mountain shape. The interpretation of the solution of linear theory for the flow over twin peaks is given in Sect. 3. Sect. 4 introduces the nonlinear numerical model used for the experiments presented in Sect. 5. A final summary and conclusion is given in Sect. 6.

2. The Orography

The superposition of two witch of Agnesi functions yields a simple mathematical expression for a 2D double bell-shaped mountain profile:

$$h(x) = h_m \left[ \frac{a^2}{a^2 + (x + x_0)^2} + \frac{a^2}{a^2 + x^2} \right]$$  \hspace{1cm} (1)

Whereas for a single witch of Agnesi profile $h_m$ and $a$ represent the true crest height and mountain half width, respectively, in the present case they only approach these values for $x_0 \to \infty$ (cf. Fig. 1 and also Fig. 2). A reduction of the separation between the peaks, $x_0$, actually increases the height $h_m^* = h_m + \Delta h_m$, and modifies the width of the two peaks, and makes the valley shallower. At the limiting case of $x_0 = 0$ the peaks merge together into one mountain with

![Fig. 1. Two single bell-shaped mountains (dotted lines) are superimposed to form the model orography (solid line) given by (1). Valley and mountain width are $x_0$ and $a$ while $h_m$ and $h_m + \Delta h_m$ are the mountain height of a single ridge and of the double bell-shaped orography, respectively](image)