A model for the horizontal exchange between ice-supersaturated regions and their surrounding area

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With 11 Figures

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Summary

A simple model is devised to simulate the turbulent exchange of humidity between ice-supersaturated regions in the upper troposphere and their subsaturated environment. The model works as a random version of a 2-dimensional cellular automaton, and does not contain dynamics nor microphysics. In spite of its simplicity the model helps to interpret certain stochastic properties of ice-supersaturated regions, namely the exponential distribution of the degree of supersaturation and the Weibull distribution of path-lengths through these regions. In particular, the exponential distribution of the degree of supersaturation evolves from permanent humidity exchange in spite of a rather peculiar initial distribution of supersaturation, that is, two Dirac delta functions. Such a robust feature is advantageous for future parameterisations of ISSRs for large-scale models.

1. Introduction

In recent years it has become quite evident that the air in the upper troposphere is often in a thermodynamic state of supersaturation with respect to ice. This fact is demonstrated by an increasing amount of humidity data obtained during various international airborne measurement campaigns where various types of hygrometer have been employed. An incomplete choice of examples can be given: POLINAT (frost point hygrometer; Ovarlez et al., 2000), SUCCESS and SONEX (Tunable Diode Laser instrument; Jensen et al., 1998; Vay et al., 2000), MOZAIIC (capacitive sensor; Helten et al., 1998, 1999). The realisation that there is often substantial ice-supersaturation in the upper troposphere is also in accordance with recent results from laboratory work on homogeneous freezing nucleation of aqueous solution droplets. Koop et al. (2000) were able to show that the freezing of such solution droplets at temperatures of less than $-40^\circ$C needs supersaturation of generally more than 40% (in order to make the solutions thin enough). The mere existence of cirrus clouds, thus demonstrates that there must be a lot of supersaturation in the upper troposphere. It is interesting to note, however, that the occurrence of ice-supersaturation is not allowed in current climate and numerical weather prediction models – an unhealthy situation which should be overcome.

Ice-supersaturated regions (ISSRs), where in spite of ice supersaturation no visible cirrus clouds could form, cover a large fraction of the sky in the upper troposphere, namely about 15% at least at northern mid-latitudes (Gierens et al., 1999). However, they are difficult to observe, and what is known so far about their properties can be reviewed in a few sentences. From an evaluation of data from the Measurement of ozone by Airbus in-service aircraft (MOZAIC) project (Marenco et al., 1998), Gierens et al. (1999) found that ice-supersaturated regions are 3–4 K
colder and have on average 50% higher absolute humidity than subsaturated regions near the tropopause. This suggests uplifting air masses (cooling) and moisture transport from lower altitudes are the essential formation processes for ISSRs. The probability of finding a certain supersaturation decreases exponentially with the degree of supersaturation. The MOZAIC data give mean supersaturations in the ISSRs of the order of 15%, which is not sufficient for the formation of visible cirrus clouds via homogeneous nucleation, but sufficient for carrying persistent contrails. Such regions are, therefore, cloud-free unless they are decorated with aircraft contrails. The MOZAIC fleet flew a mean distance (path-length) of about 150 km within ice-supersaturated areas (Gierens and Spichtinger, 2000). This mean path-length is similar to the mean path-lengths through ISSRs determined by Detwiler and Pratt (1984) who investigated the possibility for cloud seeding. Much longer path-lengths have also been found in the MOZAIC data, though rarely, e.g. on a flight from Frankfurt (Germany) to Brazil on 18 December 1995, where the flight was for about 3500 km parallel to a cold front in uplifting air masses (Gierens and Spichtinger, 2000). However, as these authors have shown, path-length statistics are subject to a severe selection bias, which leads to an over-proportional representation of large ISSRs in the data set. Therefore, the true size and shape of ISSRs, as well as the structure of ISSR fields (if these are not single isolated objects) is essentially unknown.

It is sometimes possible to conclude from the observed probability distribution of a certain physical quantity, on the nature of the underlying physical process that leads to the observed probability distribution. For example, Gierens et al. (1999) interpreted the exponential probability distribution of the degree of supersaturation in ISSRs in terms of a certain stochastic process (a so-called M/M/1 queue, see e.g. Goodman, 1988), for which turbulent humidity exchange between ISSRs and their environment would be an archetypical process. In their paper, Gierens et al. presented only a mathematical argumentation, yet they did not show a numerical simulation of the proposed process. This will be done in the present paper. Another example is the probability distribution of path-lengths through ISSRs, which can be fitted with a Weibull distribution (Gierens and Spichtinger, 2000). The Weibull distribution is a generalisation of the exponential distribution, which makes it more flexible to fit data. Hence, the Weibull distribution has been often used for modelling meteorological phenomena, including wind speed (e.g. Dixon and Swift, 1984), flood data (e.g. Boes, 1989), or rainfall intensity data (e.g. Wilks, 1989). Most recently, Berton (2000) has used the Weibull distribution to analyse cloud data of various cloud types, e.g. their ice water content, vertical extent and horizontal size. In particular, Berton (2000) found that the horizontal size of cirrus clouds could be well fitted with a Weibull distribution with an exponent $p \approx 0.5$, which is the same exponent that Gierens and Spichtinger (2000) found in the analysis of path-lengths in ISSRs. In our opinion, this agreement is not purely incidental and it points to a certain, not yet identified, relationship between ISSRs and cirrus clouds.

Unfortunately, the Weibull distribution has no simple physical interpretation (cf. Berton, 2000, sect. 7), but a relatively simple interpretation as the result of a compound process is possible: if a random variable $X$ obeys an exponential probability distribution, then a quantity $Y = aX^b$ ($a$ and $b$ constants) is Weibull distributed with an exponent $1/b$. Similarly, if the path-lengths $L$ are related via a power law $L = aX^b$ to some exponentially distributed quantity $X$, then $L$ is Weibull distributed. Hopefully simulations presented here will stimulate ideas about the so far unknown quantity $X$ and the process that couples $X$ and $L$. It is conceivable that the quantity $X$ is a measure of the initial moisture content of an ISSR when the vertical development of it comes to rest and an horizontal humidity exchange process commences. This will be further discussed in section 4.

In the following section 2 we describe the exchange model. Results are presented and sensitivities discussed in section 3. A summary is given in the final section 5.

2. The exchange model

2.1 The model and the representation of the exchange process

The exchange model runs on a two-dimensional array with toroidal topology, i.e. with periodic