The effective electroelastic property of cracked piezoelectric media

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Summary. This paper presents a study on the effective electroelastic property of piezoelectric media with parallel or randomly distributed cracks. The theoretical formulation is derived using the dilute model of distributed cracks and the solution of a single dielectric crack problem, in which the electric boundary condition along the crack surfaces is governed by the crack opening displacement. It is observed that the effective electroelastic property of such cracked piezoelectric media is nonlinear and sensitive to loading conditions. Numerical simulations are conducted to show the effects of crack distribution and electric boundary condition upon the effective electroelastic property. The transition between the commonly used electrically permeable and impermeable crack models is studied.

1 Introduction

Due to the well-known inherent coupling effects between electric and mechanical fields, piezoelectric materials have been extensively used in electromechanical devices, such as actuators, sensors and transducers. The newly developed piezoceramics are generally brittle and have a tendency to develop cracks during manufacturing and service processes. It is, therefore, essential to evaluate the coupled electromechanical behavior and the effective properties of this type of piezoelectric materials in the presence of microcracks.

The existence and development of microcracks in solid media exert important influences on various aspects of material properties. The investigation on the effective properties of materials weakened by microcracks is of great importance and has drawn significant attention from the research and industrial communities. For traditional microcracked solids where boundary conditions along the crack surfaces are well defined, various micromechanics schemes have been established to estimate the effective moduli, including the dilute model [1], [2], in which the interaction among cracks is neglected, the self-consistent method [3], [4], the differential method [5]–[7], the Mori-Tanaka method [8]–[10], and the generalized self-consistent method [11], [12]. These micromechanical models have been modified and used to study the effective property of piezoelectric materials. The effective electroelastic constants of fiber reinforced piezoelectric composites were predicted by using the self-consistent method based on a concentric cylinder model [13]. Dunn and Taya [14] estimated the effective properties of piezoelectric composites using dilute, self-consistent, Mori-Tanaka and differential micromechanics models. The effective thermo-electro-elastic moduli of multilphase fibrous composites were studied by Chen [15]. The Mori-Tanaka method was used by Dunn and Taya [16] to study the electromechanical...
properties of porous piezoelectric ceramics. Yu and Qin [17] evaluated the thermo-electroelastic properties of microcracked piezoelectric materials using the generalized self-consistent method. Dilute, self-consistent, Mori-Tanaka and differential methods are also modified to study the effective moduli of cracked thermpiezoelectric materials by Qin et al. [18].

For microcracked piezoelectric media, the electric boundary condition along crack surfaces is still one of the fundamental issues requiring further investigation. Most existing investigations in this area are based mainly on two typical crack models. One is the electrically permeable model [19], [20] and the other is the electrically impermeable model [17], [21]–[24]. These models represent two limiting cases, where the electric permittivity of the medium inside the crack is assumed to be infinite and zero, respectively. In most situations, cracks existing in piezoelectric materials are filled with dielectric media (air or vacuum), which may result in the dependence of the electric boundary condition upon the opening displacement of the crack. An elliptical crack model has been developed [25]–[28] to study the effect of the dielectric property of the crack. These studies indicate that the permeable model may underestimate the effect of the electric field on the crack initiation, while the impermeable one may overestimate its effect. It should be mentioned that in this elliptical crack model the electric boundary condition is independent of the crack deformation since only the original crack geometry is considered. For a slit crack without initial crack opening, since the dielectric constant of piezoceramics is usually 10^3 times higher than that of the dielectric medium filling the crack, the electric boundary condition may be very sensitive to the crack deformation due to the applied load. The recent study [29] for the case where the crack is perpendicular to the poling direction of the piezoelectric material indicates that when the effects of the dielectric medium inside the crack and the crack opening are considered, the problem becomes nonlinear and multiple deformation modes exist under some loading conditions. The effect of this nonlinear electric boundary condition upon an oriented crack is studied by Xu and Rajapakse [30], in which a unified formulation is provided using complex function method to account for different electric boundary conditions. Based on this dielectric crack model, Wang and Jiang [31] investigated the effective electroelastic property of piezoelectric materials containing parallel cracks perpendicular to the poling direction of the media. The result shows the significant effect of the nonlinear electric boundary condition along the crack surfaces upon the effective electroelastic property.

It is the objective of the current paper to provide a general study of the effective electroelastic property of piezoelectric materials with randomly distributed cracks. A dielectric crack model with deformation-dependent electric boundary condition and a dilute model of interacting cracks are used to determine the nonlinear behavior of the cracked medium. Numerical simulation is provided to show the effect of the electric boundary condition along crack surfaces. Attention is paid to the transition between permeable and impermeable crack models with increasing crack opening.

2 Formulation of the problem

Consider the plane problem of a piezoelectric medium weakened by randomly distributed cracks filled with a dielectric medium with negligible mechanical moduli, as shown in Fig. 1a. This piezoelectric medium is assumed to be transversely isotropic with the poling direction parallel to the y-axis of the global Cartesian coordinate system $\text{oxy}$. The medium is subjected to remotely applied external electromechanical loads $(\mathbf{t}^0, \mathbf{s}^0)$, with $\mathbf{t}^0 = \{\sigma_{22}^0, \sigma_{12}^0, D_2^0\}^T$ and $\mathbf{s}^0 = \{\sigma_{11}^0, \sigma_{12}^0, D_1^0\}^T$. A set of local Cartesian coordinate systems $\theta_k, \varphi_k, y_k \ (k = 1, 2, \ldots, N)$ is used