A novel method for solving the displacement and stress fields of an infinite domain with circular holes and/or inclusions subject to a screw dislocation

Abstract In this paper, the degenerate kernel and superposition technique are employed to solve the screw dislocation problems with circular holes or inclusions. The problem is decomposed into the screw dislocation problem with several holes and the interior Laplace problems for several circular inclusions. Following the success of the null-field integral equation approach, the typical boundary value problems can be solved easily. The kernel functions and unknown boundary densities are expanded by using the degenerate kernel and Fourier series, respectively. To the authors’ best knowledge, the angle-type fundamental solution is first derived in terms of degenerate kernel in this paper. Finally, four examples are demonstrated to verify the validity of the present approach.

1 Introduction

The subject of dislocation is essential for understanding various physical and mechanical properties of crystalline solids. Many researchers investigated dislocation problems in the past years. Smith [18] successfully solved the problem of the interaction between a screw dislocation and a circular or elliptic inclusion contained within an infinite body by using the complex-variable function and circle theorem. Besides, uniform anti-plane remote shear was also considered at the same time. Dislocation problems have been solved by using the complex variable method [1,3]. Dundurs [9] solved the screw dislocation with circular inclusion problem by using the image technique. Later, Sendeckyj [17] employed the complex-variable function in conjunction with the inverse point method to solve the problem of the screw dislocation near an arbitrary number of circular inclusions. Honein et al. [13] extended the circle theorem to solve the problem of an elastic body containing an elastic circular inclusion and subject to arbitrary loading. Sudak [19] and Jin and Fang [14] solved the problem of the screw dislocation interacting with an imperfect interface by using the complex-variable technique. Such a problem was also solved by using the image technique and Fourier transform by Fan and Wang [10]. Later, Fang and Liu [11] extended the complex-variable function and Riemann-Schwarz’s symmetry principle to solve the problem of the interaction of a screw dislocation with a circular nano-inhomogeneity incorporating interface stress. Almost all the above problems were solved by using the complex-variable technique. Its extension to three-dimensional cases may be limited. A more general approach is nontrivial for further investigation.

In this paper, we introduce the degenerate (or so-called separable) kernel for the angle-type fundamental solution ($\theta$) instead of radial-basis one ($\ln r$) to represent the screw dislocation solution. The terminology of the degenerate kernel is not coined by the authors, but refers to [2,12,16]. The degenerate kernel is also
defined in the revised manuscript by $K(x, s) = \sum_{j=1}^{\infty} A_j(s)B_j(s)$. To our best knowledge, the degenerate kernel for angle-type fundamental solution was not found in the literature. The proposed approach leads to real (as opposed to complex) equations that are free of singular integrals even when collocated on the boundary of a domain. A screw dislocation solution is decomposed into two parts. One is the screw dislocation problem with several holes, and the other is the interior Laplace problems for several circular inclusions. The interior problems can be solved by using the null-field integral approach [6]. After superimposing the two solutions, the governing equation and boundary conditions can be satisfied automatically. The present approach offers a few attractive features. First, the integrals in the boundary integral equation method (BIEM) are made simple by avoiding the senses of Cauchy and Hadamard principal values. Second, the extension of this idea to a three-dimensional problem may be straightforward. Besides, the proposed method can be seen as a kind of meshless method since no boundary element discretization is required. Finally, several illustrative examples are demonstrated to validate the present method.

2 Problem statements and mathematical formulation

The physical problem to be considered is shown in Fig. 1, where circular inclusions are imbedded in an infinite plane. For the anti-plane problem, we only consider the anti-plane displacement $w$ such that

$$u = v = 0, \quad w = w(x, y),$$

where $u$ and $v$ are the vanishing components of displacement. The governing equation for the anti-plane displacement, $w$, in the anti-plane elasticity in the absence of body force is simplified to

$$\nabla^2 w(x, y) = 0, \quad (x, y) \in D,$$

where $\nabla^2$ is the two-dimensional Laplacian operator and $D$ denotes the domain of the interest. Therefore, the screw dislocation can be described as

$$\lim_{y \to 0} [w(x, -y) - w(x, y)] = b, \quad x \geq \xi,$$

where $b$ denotes the Burgers’ vector and $\xi$ denotes the location of the screw dislocation. By taking free body along the interface between the matrix and inclusions, the problem is decomposed into two systems. One is an infinite plane with $H$ randomly distributed circular holes subject to a screw dislocation as shown in Fig. 2a. The other is $H$ randomly distributed circular inclusions bounded by contours $B_i$ ($i = 1, 2, \ldots, H$) which satisfies the Laplace equation as shown in Fig. 2b. For the problem in Fig. 2a, it can be superimposed by two parts again. One is an infinite plane subject to a screw dislocation and the other is an infinite plane with $H$ randomly distributed circular holes, which satisfies the specified boundary conditions as shown in Fig. 2c and d, respectively. The displacement arising from the screw dislocation on each boundary in Fig. 2c by using the degenerate kernel is introduced in the next section. In order to solve the interior and exterior typical boundary