The Neutron as a Quantum Object

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Neutron interference experiments elucidate various properties of matter waves and permit a novel kind of wave function tomography.

Quantum and relativity theory are still the basis for our understanding of nature. Quantum optics is part of it and contributed substantially to that understanding, due to progress in laser and quantum state preparation techniques. An extension to quantum particle optics - electrons, neutrons, atoms and molecules - makes this field of research even more attractive. In every case interference experiments yield the deepest insight into the miracles of nature.

Two decades ago the first perfect crystal neutron interferometer was tested by an Austrian-German cooperative group at the 250 kW TRIGA-reactor in Vienna [1]. Since then neutron interferometry has become a laboratory for quantum mechanical testing. Widely separated coherent beams of thermal neutrons (λ ≈ 1.8 Å, E ≈ 0.025 eV) were produced by dynamic Laue-reflection in an appropriately shaped perfect silicon crystal (Fig. 1). Analogies to the Mach-Zehnder type interferometers used in light optics, and to the Bonse-Hart interferometers [2] developed for X-rays, are obvious. Electron and the recently developed atom interferometry use somewhat similar schemes [3].

Neutrons are massive and extended quantum objects. They are Fermions with an internal structure comprising one up- and two down-quarks. Inside a magnetic field, the magnetic moment of the neutron gives rise to a two energy level system between which transitions can be induced by specific oscillating magnetic fields. Neutrons are subject to strong, electromagnetic and gravitational interactions as well as topological phenomena, which all cause measurable interference effects. Following the complementarity principle of quantum mechanics, the neutron behaves purely as a wave inside the interferometer.

It follows, from general symmetry considerations, that the wave functions originating from beam paths I and II and composing the forward beam (0) behind the interferometer are equal in amplitude and phase because they are transmitted-reflected-reflected (TRR) and reflected-reflected-transmitted (RRT), respectively. Different kinds of interaction can cause phase shifts between the coherent beams which can be calculated by the path integral of the canonical momentum k, along the interferometer loop, χ = ∫ k ds = ∆ · k.

The ideal interference pattern is therefore given by:

\[ I(\Delta) \propto |\psi(0) + \psi(\Delta)|^2 \]

\[ \propto 1 + |\Gamma(\Delta)| \cos \Delta \cdot k \]  

(1)

where Δ denotes the spatial shift of the interfering wave packets which is determined by the refractive index of the phase shifting material or field, and which also defines the phase shift χ of the interfering beams (χ = Δ · k). In the case of a pure nuclear phase shifter, one gets χ = −N · λ · D, where N denotes the particle density, D the thickness, b the coherent scattering length, and λ the wavelength of the neutron. Γ(Δ) denotes the coherence function which (as in light optics) is related to the momentum distribution function of the beam g(k).

\[ |\Gamma(\Delta)| \] approaches 1 for Δ → 0 and zero for Δ → ∞. Its characteristic spatial dimension determines the coherence length Λ, which, for Gaussian shaped beams, are directly related to the momentum distribution of the beam dκ, by the Heisenberg uncertainty relation 

\[ \Lambda^2 \delta k = \hbar/2 \]

All neutron interference experiments pertain to the domain of self-interference, where, in nearly all cases, only one neutron is inside the interferometer while the next one has yet to be born and is still contained in the uranium nuclei of the reactor fuel. Although there is no interaction between different neutrons, they have a certain common history within pre-determined limits which are defined, e.g., by the neutron moderation process, their movement along neutron guide tubes, the monochromator crystal and the special interferometer setup. Therefore, any real interference pattern contains single particle as well as ensemble properties. Minor deviations of the interference pattern from Eq. (1) may occur due to slight imperfections of the experimental setup. With a well balanced interferometer, a contrast of up to 90% can be achieved and more than the 300th order of interference can be observed. This provides the basis for various coherence experiments and for precise measurements of scattering lengths. Here we focus on more fundamental experiments, proving various intrinsic properties of quantum mechanics.

The most important experiment was probably the verification of the 4π-symmetry of spinor wave functions which was achieved independently in Europe and the U. S. soon after the successful operation of perfect crystal interferometers [4]. For the propagation of the wave function inside a magnetic field, which couples to the magnetic moment of the neutrons (H = −μB), one gets

\[ \psi(2\pi) = -\psi(0) \]

\[ \psi(4\pi) = \psi(0) \]  

(4)

which becomes measurable by the interference pattern at low order of interference.

Fig. 1: Perfect crystal neutron interferometer. The monolithic design provides the parallelism of the reflecting lattice planes to within one lattice constant. The whole system represents a macroscopic quantum device with a wide beam separation.

\[ \Gamma(\Delta) \propto \int g(\kappa) e^{i\Delta \kappa} d\kappa. \]  

(2)
\[ I \propto |\psi(0) + \psi(\alpha)|^2 \approx 1 + \cos(\alpha/2) \]  

(5)

In excellent agreement with the experimental observation (Fig. 2). The 4π-periodicity effect has been observed both for unpolarized and polarized neutrons, which demonstrates the intrinsic feature of this 4π-symmetry phenomenon and that single particle and not only ensemble properties are described by the wave function.

Extending this kind of investigation, the quantum mechanical spin-superposition law has been verified on a macroscopic scale with polarized incident neutrons split coherently and polarization-inverted in one beam path (i.e. Larmor angle rotated by \( \alpha = \pi \)), giving a wave function for the forward beam:

\[ \psi \propto \psi^I + \psi^II \propto (|z\rangle + e^{i\lambda}|-z\rangle) \]  

(6)

and a final polarization in the x,y-plane, \( P = (\cos \lambda, \sin \lambda, 0) \) i.e. perpendicular to both states before superposition [5].

Eugene Wigner pointed out that, in this case, a pure initial state in the \(|z\rangle\)-direction is transformed into a pure state in the \(|x\rangle\)-direction (\( \chi = 0 \)), although in one beam path no spin reversal (\(|z\rangle\)) and in the other a complete spin reversal (\(|-z\rangle\)) occurs.

When the spin reversal is accomplished by a Rabi-type resonance flipper, the total energy changes by the amount of the Zeeman energy \( 2\mu B_0 = h\omega_L \), causing a time-dependent phase shift (\( \omega_L \cdot t \)) and a final polarization, rotating with the Larmor frequency \( \omega_L \) in the x,y-plane without being driven by a magnetic field. A slight modification of this kind of experiment permitted the verification of the magnetic Josephson effect [6] (Fig. 3). In this case, Rabi-flippers are operated in both beam paths but with slightly different Larmor resonance frequencies, \( \omega_L^{(1)} \) and \( \omega_L^{(2)} \), due to slightly different guide fields, \( B_0^{(1)} \) and \( B_0^{(2)} \), respectively. This causes a temporal beam modulation in the form of a quantum beat effect

\[ I \propto 1 + \cos \left\{ \chi + (\omega_L^{(1)} - \omega_L^{(2)} t) \right\} \]  

(7)

where the time-dependent phase shift is driven by the magnetic interaction

\[ \Delta (t) = (\omega_L^{(1)} - \omega_L^{(2)} t) t = 2\mu AB_0 t/h. \]  

(8)

The observed period of the beam modulation and the energy sensitivity are related through the quantum mechanical uncertainty relation. The modulation is driven by an energy difference of \( \Delta E = 8.6 \times 10^{-17} \) eV and the energy sensitivity is even \( 2.7 \times 10^{-19} \) eV. This phenomenon is analogous to the well-known electric Josephson effect, where the phase shift in a superconducting tunnel junction is driven by the electric interaction \( \Delta_J(t) = 2eVt/h \).

All the phenomena described above are closely connected to the topological effects such as the geometric Berry phase [7] when the magnetic field is twisted or when the rotation is done around various axes, or the scalar Aharonov-Bohm effect when the magnetic field is switched on and off while the neutron is inside the field region [8]. This shows various additional features of the wave function which, for a long time, did not appear to be accessible by experiment. In all these cases, the canonical momentum – not the kinetic one \( (k \hbar = mv) \) – changes, and therefore no classical force acts on the particle. Figure 4 depicts this situation for charged particles (electrons) and for neutrons.

The coupling of the neutron to the gravitational field has also attracted much interest because the unification of quantum and gravitational theory is still outstanding. The action of the earth’s gravitational field appears when the interferometer crystal is rotated (angle \( \varepsilon \)) around a horizontal axis and one beam path experi-