Analysis of the packing structure of wet spheres by Voronoi–Delaunay tessellation

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Abstract The structure of a packing of narrowly sized wet spheres with packing density 0.435 is analysed against the well-established random close packing with packing density 0.64 by means of the Voronoi and Delaunay tessellation. The topological and metric properties of Voronoi polyhedra, such as the number of faces, perimeter, area and volume of a polyhedron, the number of edges, perimeter and area of a polyhedron face, have been quantified. Compared to the well-established random close packing, the distributions become wider and more asymmetric with a long tail at the higher values. The volume and sphericity of each Delaunay cell have also been quantified. Their distributions are shown to be wider and more asymmetric than those for the random close packing, but the peaks are almost the same. For the wet particle packing, the correlations between Voronoi polyhedron size and shape and between Delaunay cell size and shape are more scattered. The topological and metric results are also shown to be consistent with those obtained for the packing of fine particles, although the dominant forces in forming a packing differ. The results should be useful to the quantitative understanding of the structure of loosely packed particles.

Keywords Particle packing · Packing structure · Moisture content · Capillary force

1 Introduction

Granular matter is widely encountered in nature and in engineering practice, and has been subjected to intensive research (see [1–6] for example). One of its key fundamental areas is the quantification of the structure of particle packing. Mainly driven by the need to model liquids [7], some experimental efforts have been made to establish such information for cohesionless uniform spheres where the dominant force is the gravity [8–11]. The information has been widely used in the structural modelling of simple liquids, glasses and amorphous solids, and transport properties of porous media [12–24].

Forming a packing is a dynamic process involving various interparticle forces in addition to the gravity force, including the capillary force associated with wet particles and the van der Waals force with fine particles [25,26]. The presence of these cohesive forces will significantly change the static and dynamic behaviour of particles, which has become a research focus in granular research in the past decade or so [27–46]. While steady progress is being made in recent years in developing a macroscopic understanding of the packing of cohesive particles, to date no measured structural information is available. To overcome this gap, recently we successfully measured the coordinates of spheres in a wet packing system by means of optical microscopy method [47]. From the information obtained, the wet packing structures in terms of coordination number and radial distribution function have been analysed. The results confirm that the presence of strong cohesive forces among particles results in the formation of agglomerates, large pores and chain-connected particles in a packing.

The analysis of the wet packing structures can be extended by means of Voronoi and Delaunay tessellation. Since the work of Bernal and Finney [11,12], Voronoi and Delaunay
tessellation has been widely used in the analysis of the packing structure of particles [49–52,55]. Voronoi and Delaunay analysis can provide detailed information such as the distributions of volume, surface area, numbers of faces and pore size. For a random packing, a Voronoi polyhedron (VP) is a complex polyhedron, and its VPs describe the arrangement of all the neighbours of a particle. On the other hand, a Delaunay simplex (DS) is a disordered tetrahedron representing the structure of clusters composed of four adjacent particles in a packing. Both VP and DS have been directly used in engineering application. For example, VP has been found to be useful to quantify the properties related to the connectivity of particles, e.g. effective thermal conductivity [23], while DS is particularly useful to study the properties related to the connectivity of pores, e.g. the transport phenomena [20–22]. Recently Yang et al [45,56] investigated the VP and DS properties of the packing of fine particles. The packings were built by means of discrete particle simulation in which the dominant force is the van der Waals force. Their work shows that the VP properties heavily depends on packing density.

In this work, we apply the Voronoi-Delaunay tessellation to analyze the packing of wet particles and quantify the topological and metric properties of Voronoi polyhedra, and the volume and sphericity of each Delaunay cell. For comparison, Voronoi-Delaunay analysis is also applied to the random close packing [11]. The structural results of the wet packing are also compared with those for fine particles [45,46].

2 Experimental work

The packing experiment performed mainly involves three steps: mixing, packing and measuring, as described elsewhere [33,48]. To obtain the liquid content, the packing was weighed, and re-weighed after drying. The measured weights, together with the particle and liquid densities, allow the calculation of liquid content, expressed as the volume ratio between liquid and particles. The ratio of the volume of particles to the volume they occupy is used as a measure of the packing behaviour, and referred to as dry based packing density (packing density hereafter). Such concepts have been widely used [29,33,42,46–48]. It has been reported that light particles are more sensitive to liquid addition [33]. Therefore, to highlight the effect of capillary force to a maximum extent and increase the precision of measurement, expanded polystyrene beads of specific density 0.136 are used in this work. Their sizes are uniformly distributed in a narrow range of 5.3 to 6.2 mm, with the mean size being 5.75 mm. Optical observation demonstrates the selected beads are reasonably spherical (sphericity is greater than 0.97). The container used is 255 mm in diameter and 210 mm in height. After many trials, the liquid used is a dilute glue solution of specific density 0.995, viscosity $6.01 \times 10^{-3}$ Pa s, and surface tension $47.6 \times 10^{-3}$ N m$^{-1}$. It can produce a packing with enough adhesive strength allowing a packing sample to be disassembled one particle at a time without changing the remains.

The coordinates of particles are measured by means of a TM-500 digital microscope with a three dimensional mechanical stage manufactured by Mitutoyo Corporation. The precision is within $\pm 0.05$ mm in vertical direction and $\pm 0.01$ mm in horizontal direction. This produces a standard error within 0.8% of particle diameter as compared to 1.0 and 0.21%, respectively for the work of Scott [9] and Bernal et al. [10]. To avoid the wall effect, the packing sample is taken from the central part of a packing, at least five particle diameters away from the wall, and the top and bottom surfaces of the packing. The coordinates and size of each particle in the sample are measured one by one and stored in a computer to reproduce the packing structure for analysis.

To verify the present measuring technique, one experiment is also made for the packing of dry particles that is prepared under the random dense packing conditions. The particles are first slowly poured into a calibrated container through a funnel. Then the packing is gently vibrated in vertical direction to obtain the achievable maximum packing density. Then the diluted glue solution is introduced into the packing from the bottom and drained. During these operations, the particles are capped to avoid variation in structure. After drying, the structure of the packing is measured by the technique described above. The data produced are compared with those established in the literature [11].

3 Results and discussion

3.1 General characteristics

Figure 1 shows the measured relationship between dry based density and liquid content. It can be seen that the packing density decreases to a minimum and keeps almost constant before increasing as the liquid content increases. The results are qualitatively comparable with those reported for glass beads [33]. However, the density values are much lower, mainly because of the difference in particle density. The lower the density ratio between particle and liquid, the lower the density for a given liquid content [33]. As mentioned earlier, particles of low density have been purposely selected for this work. The structure of the packing when the liquid content is 2.5% is then measured using the technique described above. This involves the measurements of the coordinates and diameters of 3801 particles. The overall density of this packing is 0.417 according to Fig. 1 or 0.435 for the structural sample as a result of excluding the wall effect, much lower than 0.64 for the random close packing (RCP) [11,12].

The validity of the present experimental technique was examined for the packing of dry particles, obtained under