Mutual Action of Optical Activity and Electro-Optic Effect and Its Influence on the Electro-Optic Q-switch

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In this paper, we have studied the mutual action of the optical activity and the electro-optic effect, and its influence on the electro-optic Q-switch. The birefringence \((g_{33}/n_0)(\Delta/n_0)^2\) resulting from the mutual action is calculated. In the polarization light interferometric experiment setup, the outgoing light intensity expression is given by

\[
I = A_0^2 \cos^2 \left[ \frac{\pi}{\lambda} (g_{33}/n_0) l - \beta \right] + \left( \pi/\lambda \right) \left[ (-2\Delta) + (g_{33}/n_0)(\Delta/n_0)^2 \right],
\]

for the optically active crystal on which the voltage is applied.

We discuss the influence on the turn-off and turn-on states of the Q-switch caused by the mutual action term \((g_{33}/n_0)(\Delta/n_0)^2\), and the advantage and disadvantage of the two work-manners of the Q-switch (i.e., step-up manner and step-down manner). The synthetic properties of \(La_{3}Ga_{5}SiO_{14}\) electro-optic Q-switch are described in comparison to KD\(P\)O\(_4\) and Li\(Nb\)O\(_3\) Q-switches. © 2004 The Optical Society of Japan

Key words: optical activity, electro-optic property, electro-optic Q-switch, \(La_{3}Ga_{5}SiO_{14}\) single crystal

1. Introduction

From a traditional point of view, it is complicated to make electro-optic devices using crystals which are optically active. The authors of the “Handbook of Optics” state:1)

“The interest in crystalline Quartz comes from its availability and excellent optical properties rather than from a large electro-optic effect. Crystals of Quartz are optically active, and this complicates the use of the material as an electro-optic modulator.” This statement comes after their comment that (NH\(_4\)H\(_2\)PO\(_4\)), (KH\(_2\)PO\(_4\)), (KD\(P\)O\(_4\)), (KNbO\(_3\)), (Li\(Nb\)O\(_3\)), (Li\(Ti\)O\(_3\)), (Ba\(Ti\)O\(_3\)) and (Sr\(Ba_{1-x}\) Nb\(_2\)O\(_4\)) electro-optic crystals.

They are not willing to set foot in that research field. Recently, a new electro-optic Q-switch was developed in our laboratory using \(La_{3}Ga_{5}SiO_{14}\) (abbreviated as LGS\(^2\)) single crystal, which is optically active.\(^3\)

This type of Q-switch behaves just as those do which are not optically active. It can be used in medium-energy output lasers to take the place of DK\(P\)O\(_4\) Q-switch. Therefore, it is necessary to study the mutual action of the optical activity and the electro-optic effect, and its influence on the electro-optic Q-switch.

2. Mutual Action of Optical Activity and Electro-Optic Effect

2.1 Mutual action of optical activity and electro-optic effect

From the electric and magnetic fields theories of optical activity,\(^5\) we know that when a plane-polarization light with the amplitude \(A_0\) travels through an optically active crystal (which is uniaxial crystal) along its z axis, it resolves two circular polarization lights, one is left-rotation and the other is right-rotation. The two circular polarization lights have the same frequency \(\omega\), the same amplitude \(A_0/\sqrt{2}\) and the same initial phase \(\varphi\). The refractive indices of these two lights are expressed as follows\(^5\)

\[
\begin{align*}
\ni_l &= n_0 + \frac{g}{2n_0}, \\
\ni_r &= n_0 - \frac{g}{2n_0},
\end{align*}
\]

where \(\ni_l\) and \(\ni_r\) are the refractive indices of the left-rotation circular polarization light and the right-rotation circular polarization light, respectively, \(g\) is the relative branch quantity of optically active tensor, and \(n_0\) is the refractive index of the ordinary light of the uniaxial crystal.

If the crystal is biaxial, the plane polarization light will resolve two elliptical polarization lights, and the refractive indices of these two elliptical polarization lights become\(^5\)

\[
\begin{align*}
\ni_l &= n_1 + \frac{g}{2n_1}, \\
\ni_r &= n_2 - \frac{g}{2n_2},
\end{align*}
\]

where \(n_1, n_2\) are the refractive index of x axis direction and that of y axis direction, respectively, \(\ni_l\) and \(\ni_r\) are the refractive indices of the left-rotation ellipse polarization light and the right-rotation ellipse polarization light, respectively.

Now we take LGS crystal as an example to study the mutual action of the optical activity and the electro-optic effect.

\(La_{3-1}Nd_{x}La_{3}Ga_{5}SiO_{14}\) (Nd\(^{3+}\):LGS) single crystal was first investigated by A. A. Kaminskii et al. as a type of laser material in 1983.\(^2,3\) LGS single crystal belongs to the trigonal system, 32 point group and spatial group P\(3\)21. The gyration tensor of LGS single crystal is\(^5\)

\[
\begin{pmatrix}
g_{11} & 0 & 0 \\
0 & g_{22} & 0 \\
0 & 0 & g_{33}
\end{pmatrix}
\]

and the matrix of the electro-optic coefficients is

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\[
\begin{pmatrix}
\gamma_{11} & 0 & 0 \\
-\gamma_{11} & 0 & 0 \\
\gamma_{41} & 0 & 0 \\
0 & -\gamma_{41} & 0 \\
0 & 0 & -\gamma_{11}
\end{pmatrix}.
\]

There are two independent electro-optic coefficients \(\gamma_{11} = -\gamma_{21} = -\gamma_{02}, \gamma_{41} = -\gamma_{52}\) in the matrix, and they have been determined by the interferometric method. The results obtained are: \(\gamma_{11} = 2.2 \text{ pm/V}, \gamma_{41} = 1.8 \text{ pm/V}\). These results are a little different from those in ref. 7.

If an electric field is applied on LGS along its x axis, it will be biaxial in optical property. The change of the refractive index \(dn\) induced by the electro-optic effect is given by\(^5\)

\[dn = -\frac{1}{2} n_0^3 \gamma_{11} E_1.\]  (3)

For simplification of writing, we define \(\Delta = \frac{1}{2} n_0^3 \gamma_{11} E_1\).

Substituting \(n_1 = n_0 - \Delta\), \(n_2 = n_0 + \Delta\) and \(g = g_{33}\) into eq. (2), we can obtain the refractive index expression of the two elliptical polarization lights resulting from the electro-optic effect

\[
\begin{align*}
n_l &= (n_0 - \Delta) + \frac{g_{33}}{2(n_0 - \Delta)} \\
&= (n_0 - \Delta) + \frac{g_{33}}{2n_0(1 - \Delta/n_0)} \\
n_r &= (n_0 + \Delta) - \frac{g_{33}}{2(n_0 + \Delta)} \\
&= (n_0 + \Delta) - \frac{g_{33}}{2n_0(1 + \Delta/n_0)}.
\end{align*}
\]  (4)

This is true because \(\Delta/n_0 \ll 1\), eq. (4) can be expanded as Taylor series for \(\Delta/n_0\)

\[
\begin{align*}
n_l &\approx (n_0 - \Delta) + \frac{g_{33}}{2n_0} \left[1 + \frac{\Delta}{n_0} + \left(\frac{\Delta}{n_0}\right)^2 + \cdots \right] \\
n_r &\approx (n_0 + \Delta) - \frac{g_{33}}{2n_0} \left[1 - \frac{\Delta}{n_0} + \left(\frac{\Delta}{n_0}\right)^2 - \cdots \right].
\end{align*}
\]  (5)

After the two ellipse polarization lights pass through LGS crystal, their amplitudes compose a vector. According to the amplitude vector superposition principle,\(^8\) the amplitude of vector \(A\) can be expressed as

\[A^2 = \left(\frac{A_0}{2}\right)^2 + \left(\frac{A_0}{2}\right)^2 + 2\left(\frac{A_0}{2}\right)\left(\frac{A_0}{2}\right) \cos \Gamma\]

\[= A_0^2 \cos^2 \frac{\Gamma}{2},\]

where \(\Gamma\) is the phase difference caused by the birefringence of the two ellipse polarization lights.

From eq. (5), \(\frac{\Gamma}{2}\) can be calculated by

\[
\frac{\Gamma}{2} = \frac{\pi}{\lambda} (n_l - n_r) l \\
= \frac{\pi}{\lambda} \left(\frac{g_{33}}{n_0}\right) l - \frac{\pi}{\lambda} (2\Delta) l + \frac{\pi}{\lambda} \left(\frac{g_{33}}{n_0}\right) \left(\frac{\Delta}{n_0}\right)^2 l,\]  (7)

where \(\lambda\) is the wavelength of the light, and \(l\) is the length of the sample along the optical axis direction.

Now, we discuss the outgoing light intensity in the polarization light interferometric experiment setup.

### 2.2 Outgoing light intensity expression of the optically active crystal to which no voltage is applied

After passing through LGS crystal, the light travels through an analyzer (see Fig. 1).

Figure 1 shows the polarization light interferometric experiment setup for the optically active crystal. If no electric field is applied to LGS crystal (i.e., \(\Delta = 0\)), the intensity of the outgoing light (which passes through the analyzer) \(I\) can be expressed as follows

\[I = A_0^2 \cos^2 \alpha,\]  (8)

where \(\alpha\) is the angle between the analyzer and the polarized direction of the light which passes through the optically active crystal.

With reference to the x axis, the relation of \(\alpha\) and other angles is shown in Fig. 2.

\[\beta\] is the angle from the polarizer (parallel to x axis) to the polarized direction of the light which passes through the LGS crystal, \(\Phi\) is the angle from the polarizer to the analyzer. \(\Phi\) is given by

\[\Phi = \frac{\pi}{\lambda} \left(\frac{g_{33}}{n_0}\right) l = \rho l,\]  (9)

where \(\rho\) is called the rate of optical activity.\(^5\) For a 1.642 \(\mu\)m wavelength, this is equal to 1.2°/mm. \(\alpha\) is the angle from the

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**Fig. 1.** Experimental setup of the polarization light interference for optically active crystal LGS.

**Fig. 2.** Relation between \(\alpha\) and \(\Phi, \beta\) with reference to the x axis.