Shift, Rotation and Limited Scale Invariant Pattern Recognition Using Synthetic Discriminant Functions

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Making use of the idea of synthetic discriminant functions (SDF), a simple way to synthesize a shift, rotation and limited size correlation filter is proposed. The SDF is synthesized by superimposing four 2nd order circular harmonics of a training reference pattern in 4 different sizes. Computer simulation experiments have shown that the filter is indeed shift, full rotation and limited size invariant over a size range from 1 to 1.82. The invariant range can be increased if more training patterns are used.

Key words: pattern recognition, shift, rotation and limited size invariant, synthetic discriminant functions, circular harmonic function

1. Introduction

Since Vander Lugt first developed the matched spatial filter (MSF) for coherent pattern recognition in the sixties, people have tried to find a filter that is insensitive to rotation and scaling of the target. During the last several decades, some ideas have been proposed which partially solved this problem. Casasent and Psaltis proposed a solution to the scale invariant problem using optical Mellin transform in 1976. Hau and Arsenault solved the shift and rotation invariant problem using the circular harmonic filter (CHF) in 1982. Mendlovic et al. and Rosen and Shamir solved the shift and scale invariant problem in 1988 and 1989, respectively. Saito et al. and Cassent et al. proposed real time scale and rotation invariant optical correlators using a method of coordinate transformation in 1983 and 1987, respectively.

Casasent and Psaltis proposed a hybrid (optical and electronic) correlator which is capable of performing shift, rotation and scale invariant pattern recognition using coordinate transformation in 1976. However, the correlation output of this system gives rise to a twin-peak when the input pattern matches the reference pattern. Mersereau and Morris proposed in 1986 a white light illumination correlator using CHF, which is shift, rotation and scale invariant. The correlation process of this correlator is, however, cumbersome since selection of proper wavelength is needed in the operation.

In 1980, Hester and Cassent proposed the idea of synthetic discriminant functions (SDF) for pattern recognition and pattern classification. Making use of the idea of SDF, a correlator filter possessing shift, rotation and limited scale invariance of a certain range can easily be designed. According to the theory of SDF, the desired synthetic discriminant function is synthesized by adding up several mathematical functions algebraically, each of which is derived from a certain training pattern. If the training patterns are taken from the various versions of a certain reference pattern in different rotational configurations and different sizes, the resulting SDF filter will be limited rotation and limited scale invariant. The range of invariance, of course, depends on the number of training patterns. The more training patterns used, the larger the range of the invariance will be. However, the more training patterns used, the more calculation is needed to obtain the synthetic discriminant function and hence more computer time and a more sophisticated computer is required. In addition to that, as the number of training patterns is increased, the signal-to-noise ratio (SNR) decreases.

Instead of constructing the desired SDF filter by linearly superimposing component functions representing, respectively, the full picture of training patterns, we propose to fabricate the filter by adding up circular harmonics of the same order of the training patterns. The training patterns are a single reference pattern in different sizes, the resulting filter will be limited scale invariant but rotation invariant, because circular harmonic filters are rotation invariant. The filter is to be used in a 4f coherent correlator, it is therefore shift invariant.

In this article, we use the English letter ‘E’ as the reference pattern or target, and four reference patterns in relative sizes 1.09, 1.27, 1.45 and 1.65 are chosen for the fabrication of the desired SDF. This set of patterns also serves as the training patterns. Experimental results have shown that the filter is indeed shift, rotation and limited scale invariant. The range of scale invariance is found to be from 1.00 to 1.82. The range of this invariance can, of course, be increased if more reference patterns of a large scaling range are to be used.

2. Fabrication of the SDF

Let $f(r, \theta)$ be the complex amplitude of the reference pattern of unit relative size expressed in a polar coordinate system. According to the theory of circular harmonics,

$$ f(r, \theta) = \sum_{m=-\infty}^{\infty} f_m(r) \exp(i m \theta), \quad (1) $$

where $m$ is an integer, and
where \( f_m(r) \exp(i m \theta) \) is called the \( m \)th order circular harmonic of \( f(r, \theta) \). If the size of the reference pattern is changed by a linear scaling factor \( a \), the pattern is expressed as \( f(r/a, \theta) \).

In this paper, the desired SDF \( h(r, \theta) \) is synthesized by adding up linearly the circular harmonics of the same order of \( N \) different sized reference patterns. The function \( h(r, \theta) \) therefore assumes the following form:

\[
h(r, \theta) = \sum_{k=1}^{N} A_k f_{\kappa}(r/a_k, \theta) \exp(i m \theta)
\]

where \( f_{\kappa}(r/a_k \theta) \exp(i m \theta) \) is the \( m \)th order circular harmonic of the \( k \)th reference pattern, \( a_k \) being the scaling factor. \( \{A_k\} \) is a set of unknowns to be determined, they being the complex weighting factors associated with \( f_{\kappa}(r/a_k \theta) \exp(i m \theta) \); \( N \) is the number of training patterns. In this paper four training patterns are used, thus \( N=4 \). Second order circular harmonics of the reference patterns have been chosen to synthesize the \( h(r, \theta) \), thus \( m=2 \).

Since the proposed SDF filter contains the factor \( \exp(i m \theta) \), rotation of the input patterns about the origin of the input plane does not affect the correlation intensity at the origin of the output plane. The filter is therefore rotation invariant. On the other hand, since the filter is used in a 4f coherent correlator, it is shift invariant. As a result, the proposed filter is both shift and rotation invariant. Accordingly, it suffices in the following discussion to consider the respective responses of the filter to the set of training patterns, \( f(r/a_k \theta) \), centered at the input plane. We would like to mention that in our analysis, after the proper centering of the reference pattern has been chosen as the origin of the coordinate system. Therefore, output intensity peaks occur at the origin of the output plane when the input patterns are centered at the input plane (i.e., when the position of the input patterns is such that their proper center is at the coordinate origin of the input plane).

It should be noted that, in the following analysis, the positive directions of the coordinate axes of the input plane, Fourier transform plane and the output plane are the same. Accordingly, when the \( n \)th training pattern is presented to the input plane of the correlator, the output amplitude distribution \( C_n(x,y) \) will be

\[
C_n(x,y) = f\left(\frac{r}{a_n}, \theta\right) \ast h(x,y)
\]

where the symbol \( \ast \) denotes the correlation integral for which an expression such as \( g(x,y) \ast h^*(x,y) \), in fact, stands for

\[
\int g(\xi, \eta) h^*(\xi + x, \eta + y) d\xi d\eta
\]

In the above equations and expressions, the symbol \( ^* \) in the upper right corner of a quantity denotes the complex conjugate of that quantity. Substituting Eq. (3) in Eq. (4) yields

\[
C_n(0,0) = \sum_{\kappa=1}^{N} A_{\kappa} C_{\kappa}(0,0) = \exp(i \phi_0)
\]

where the \( C_{\kappa}(0,0) \)'s can be calculated from Eq. (6). In Eq. (7), both \( \{A_{\kappa}\} \) and \( \{\phi_0\} \) are unknowns to be determined. Once \( \{A_{\kappa}\} \) is determined, the desired SDF can be synthesized according to Eq. (3).

The value of the \( \{A_{\kappa}\} \) and \( \{\phi_0\} \) is determined with an electronic computer according to the following algorithm. A set of values for the \( \{\phi_0\} \) is chosen first, then the \( \{A_{\kappa}\} \) for this set of \( \{\phi_0\} \) is calculated from Eq. (7). Next, the whole correlation output distribution for each training pattern is calculated by Eq. (5). The four output correlation distributions for the four training patterns are then examined, the highest sidelobe in these four distributions is located and the value of it is stored. At this point, the first cycle for the determination of \( \{A_{\kappa}\} \) and \( \{\phi_0\} \) is finished. After that, another set of values of \( \{\phi_0\} \) is selected and the above steps repeated. Similar cycles consisting of these calculations, examinations and storing are carried out until all possible combinations of the \( \{\phi_0\} \) are exhausted. After all the highest sidelobes for the various sets of \( \{\phi_0\} \) are stored, they are compared and the minimum is determined. The set of \( \{\phi_0\} \) corresponding to this minimal highest sidelobe is our actual \( \{\phi_0\} \) used to determine the \( \{A_{\kappa}\} \) and subsequently the synthesis of the desired SDF. Obviously this SDF has the best peak to sidelobe ratio (PSR). In the process of the determination of the \( \{\phi_0\} \) and \( \{A_{\kappa}\} \), only one of the four \( \phi_0 \) values is changed in each cycle, and its variation is \( 1.8' \).

3. Experimental Results

After the desired SDF was synthesized, computer simulation was carried out using it in a 4f coherent correlator to identify the reference pattern ‘E’ of various sizes and in various rotational configurations. Figure 1(a) shows the various input reference patterns of which B, D, F, and H