Bose–Einstein effect in $Z^0$ decay and the weight method

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Received: 23 September 1999 / Published online: 3 February 2000 – © Springer-Verlag 2000

Abstract. We discuss the Bose–Einstein interference effect in multi-particle production. After a short review of various methods of implementation of this effect into Monte Carlo generators the weight method is presented in more detail and used to analyze the data for hadronic $Z^0$ decays. In particular, we consider the possibility of deducing the two-particle weight factor from the experimental data.

1 Introduction

In the last years many papers have been devoted to experimental and theoretical studies of Bose–Einstein interference effects \cite{1} in multi-particle production. It has been argued that these studies may allow for the reconstruction of the space-time development of the interactions. In particular, different possibilities of implementing interference effects into Monte Carlo generators used for high energy hadroproduction processes were discussed. In this note we add some points to this discussion.

We present in detail some aspects of the weight method of implementing the BE effect in the MC generators. In particular, we try to establish to what extent one may reconstruct the two-particle weight function (related to the Wigner function) from the data on the BE effect. For this purpose we use the data on multi-particle production with the highest statistics available, the hadronic decays of $Z^0$'s produced in $e^+e^-$ collisions.

A short summary of various methods of implementing interference effects into Monte Carlo generators is presented in the next section. In the Sect. 3 we discuss the data from different LEP experiments on two-particle correlations from $Z^0$ hadronic decays which were used for analyzing the Bose–Einstein effect. Sect. 4 is devoted to the analysis of data in terms of the weight model. In particular, various choices of two-particle weight factors used in this method are compared. The last section contains some conclusions and an outlook for further investigations.

2 Implementation methods

The standard discussion of the BE effect in multi-particle production \cite{2} starts from the classical space-time source emitting identical bosons with known momenta. Thus, the most natural procedure is to treat the original Monte Carlo generator as the model for the source and to symmetrize the final state wave function \cite{3}. This may be done in a more proper way using the formalism of Wigner functions \cite{4}. In any case, however, the Monte Carlo generator should yield both the momenta of the produced particles and the space-time coordinates of their creation (or last interaction) points. Even if we avoid troubles with the uncertainty principle by using the Wigner function approach, such a generator seems reliable only for heavy ion collisions. It has been constructed also for the $e^+e^-$ collisions \cite{5}, but localizing the hadron creation point in the parton-based Monte Carlo program for lepton and/or hadron collisions is a rather arbitrary procedure, and it is hard to say what one really tests comparing such a model with the data.

It seems to be the best procedure to take into account the interference effects before generating events. Unfortunately, this was done until now only for the JETSET generator for a single Lund string \cite{6-9}, and a generalization for multi-string processes is not obvious. No similar modifications were yet proposed for other generators.

The most popular approach, applied since years to the description of BE effect in various processes, is to shift the final state momenta of the events generated by the PYTHIA/JETSET generators \cite{10,11}. The prescription for a shift is such as to reproduce the experimentally observed enhancement in the ratio

$$c_2(Q) = F \frac{\int d^3p_1d^3p_2\rho_2(p_1,p_2)\delta(Q-\sqrt{-(p_1-p_2)^2})}{\int d^3p_1d^3p_2\rho_1(p_1)p_2(p_2)\delta(Q-\sqrt{-(p_1-p_2)^2})},$$

where $F$ is a normalization factor:

$$F = \langle n \rangle^2/\langle n(n-1) \rangle.$$  

The function $c_2(Q)$ depends on a single invariant variable $Q$. The value of this function is close to one for the default JETSET/PYTHIA generator. One often parameterizes this ratio by

$$c_2(Q) = 1 + \lambda \exp(-R^2Q^2),$$

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where \( R \) and \( \lambda \) are parameters interpreted as the source radius and the “incoherence strength”, respectively.

After performing the shifts, all the CM 3-momenta of the final state particles are rescaled to restore the original energy. In more recent versions of the procedure [12] “local rescaling” is used instead of the global one. In any case, each event is modified and the resulting generated sample now exhibits “BE enhancement”: the ratio (1) is no longer close to one, and it may be parameterized as in (2).

There is no theoretical justification for this procedure, so it should be regarded as an imitation rather than implementation of the BE effect. Its success or failure in describing data is the only relevant feature. Unfortunately, whereas the method is very useful for the description of two-particle inclusive spectra, it fails to reproduce (with the same fit parameters \( R \) and \( \lambda \)) the three-particle spectra [13] and the semi-inclusive data [14]. This could certainly be cured, e.g., by modifying the shifting procedure and fitting the parameters separately for each semi-inclusive sample of data. However, the fitted values of parameters needed in the input factor (2) used to calculate shifts are quite different from the values one would get fitting the resulting ratio (1) to the same form [15]. This was shown recently in a much more detailed study [16]. Thus, it seems to be very difficult to learn something reliable of the space-time structure of the source from the values of the fit parameters in this procedure.

All this has lead to a revival of weight methods, which have been known for quite a long time [17, but were plagued with many practical problems. The method is clearly justified within the formalism of the Wigner functions, which allows one to represent (after some simplifying assumptions) any distribution with the BE effect built in as a product of the original distribution (without the BE effect) and the weight factor, depending on the final state momenta [18]. With the extra assumption of factorization in momentum space, we may write the weight factor for the final state with \( n \) identical bosons as

\[
W(p_1, \ldots, p_n) = \sum_{i=1}^{n} w_2(p_i, p_{P(i)}),
\]

where the sum extends over all permutations \( P_n(i) \) of \( n \) elements, and \( w_2(p_i, p_k) \) is a two-particle weight factor reflecting the effective source size. A commonly used simple parameterization of this factor for a Lorentz symmetric source is

\[
w_2(p, q) = \exp((p - q)^2/2Q_0^2).
\]

The only free parameter now is \( Q_0 \), representing the inverse of the effective source size. In fact, the full weight given to each event should be a product of factors (3) calculated for all kinds of bosons: in practice, pions of all signs are counted. Only direct pions and the decay products of \( \rho, K^* \) and \( \Delta \) should be taken into account, since for other pairs a much larger \( R \) should be used, resulting in negligible contributions.

The main problem of the weight methods is that weights do change not only the Bose–Einstein ratio (1), but also many other distributions. Thus, with the default values of the free parameters (fitted to the data without weights) we inevitably find some discrepancies with the data after introducing weights.

We want to make clear that this cannot be taken as a flaw of the weight method. There is no measurable world “without the BE effect”, and it makes not much sense to ask if this effect changes e.g. the multiplicity distributions. If any model is compared to the data without taking the BE effect into account, the fitted values of its free parameters are simply not correct. They should be refitted with weights, and then the weights recalculated in an iterative procedure. This, however, may be a rather tedious task.

Therefore, we use a simple rescaling method proposed by Jadach and Zalewski [20]. Instead of refitting the free parameters of the MC generator, we rescale the BE weights (calculated according to the procedure outlined above) with a simple factor \( cV^n \), where \( n \) is the global multiplicity of “direct” pions, and \( c \) and \( V \) are fit parameters. Their values are fitted to minimize

\[
\chi^2 = \sum_n \left( cV^n N^w(n) - N^0(n) \right)^2 / N^0(n),
\]

where \( N^0(n) \) is the number of events for the multiplicity \( n \) without weights, and \( N^w(n) \) is the weighted number of events. This rescaling restores the original multiplicity distribution [24]. In addition, the single longitudinal and transverse momentum spectra are also restored by this rescaling [24].

Obviously, for a more detailed analysis of the final states, single rescaling may not be enough. For instance, since different parameters govern the average number of jets and the average multiplicity of a single jet, both should be rescaled separately to avoid a discrepancy with the data. Let us stress once again that such problems arise due to the use of generators with improperly fitted free parameters, and do not suggest any flaw of the weight method. Another problem is that our formula for the weights (3) is derived using some approximations, which are rather difficult to control [18]. We can justify them only a posteriori from the phenomenological successes of the weight method.

Last but not least, the main practical difficulty with (3) is the factorial increase of the number of terms in the sum with increasing multiplicity of identical pions \( n \). For high energies, when \( n \) often exceeds 20, a straightforward application of (3) is impractical [19], and some authors [20,21] replaced it with simpler expressions, motivated by some models. It is, however, rather difficult to estimate their reliability.

We have recently proposed two ways of dealing with this problem. One method consists of a truncation of the sum (3) up to terms, for which the permutation \( P(i) \) moves no more than five particles from their places [22]. However, it is difficult to claim a priori that such a truncation does not change the results which would be obtained using the full series (2).

Therefore a second way of an approximate calculation of the sum (2) was proposed [23]. Since this sum, called