The components of the $\gamma^*\gamma^*$ cross section

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Abstract. We extend our previous treatment of the $\gamma^* p$ cross section based on Gribov’s hypothesis to the case of photon–photon scattering. With the aid of two parameters, determined from the experimental data, we separate the interactions into two categories corresponding to short (“soft”) and long (“hard”) distance processes. The photon–photon cross section thus receives contributions from three sectors, soft–soft, hard–hard and hard–soft. The additive quark model is used to describe the soft–soft sector, pQCD the hard–hard sector, while the hard–soft sector is determined by relating it to the $\gamma^* p$ system. We calculate and display the behaviour of the total photon–photon cross section and its various components and polarizations for different values of energy and virtuality of the two photons, and discuss the significance of our results.

1 Introduction

Scattering in the high energy (low $x$) limit has been studied in perturbative QCD (pQCD) over the past few years, mainly through the analysis of deep inelastic (DIS) events of lepton–hadron and hadron–hadron collisions. Such pQCD investigation requires some knowledge of the non-perturbative contribution which is introduced through the initial input to the evolution equations or put in explicitly. In this paper we present a study of virtual photon–photon scattering. Our investigation is based on our model for $\gamma^* p$ cross section \cite{1}, which provides the framework for the present calculation. Our goal is twofold.

1. In any QCD process, finding the dynamics for intermediate distances is still an open problem, as it involves a transition between short distance (“hard” - perturbative) and large distance (“soft” - non-perturbative) physics. In \cite{1} we have suggested a procedure, based on Gribov’s general approach \cite{2}, of how to accommodate both contributions in DIS calculations. Two photon physics involves an obvious reaction where these ideas can be further studied and re-examined.

2. Virtual photon–photon scattering has been proposed \cite{3–6} as a laboratory to study the BFKL Pomeron \cite{7}, as the total cross section of two highly virtual photons provides a probe of BFKL dynamics. Our study enables one to estimate the background to the proposed BFKL process. This background consists of two contributions: (i) We give an explicit estimate of the soft component in $\gamma^*\gamma^*$ scattering. (ii) Our pQCD estimate for the hard component is based on DGLAP \cite{8} and as such can be used to assess when the BFKL dynamics starts to dominate.

Impressive attempts have been made \cite{9,10} to describe two photon physics within the framework of the vector dominance model (VDM) mainly as a soft interaction. However, one can consider a two photon interaction as an interesting tool for investigating the interplay between soft and hard physics \cite{11}. The photon can appear as an unresolved object or as a perturbative fluctuation into an interacting quark–antiquark system. A careful analysis of the various components of the total cross section will help us understand the interface of the short distance and large distance interaction.

In $e^+ e^-$ colliders, the measurement of the $\gamma^*\gamma^*$ is carried out by double tagging the outgoing leptons close to the forward direction, as most of the initial energy is taken by the scattered electrons. The double tagged cross section falls off with the increase of the photons’ virtualities due to the photon propagator. The experimental statistics are improved for single and no tag events where one of the colliding photons or both are quasi-real \cite{12}. There is, therefore, a theoretical interest and an experimental need to better understand and estimate the perturbative and non-perturbative contributions with realistic configurations of the two photon virtualities.

Our paper deals with photon–photon collisions in the high energy limit, which confines us to low $x$ values. A pQCD investigation of $e\gamma$ DIS is non-trivial \cite{13} due to the dual nature of the photon target (quasi-real or virtual) which can be perceived as either a hadron-like partonic system or a point-like object. The resulting difficulties in pQCD calculations of $F_2^\gamma$ in the small $x$ limit have been extensively discussed in the literature and several strategies have been devised to bypass these problems \cite{13}. For the
purpose of our analysis we follow the approach suggested by Glick and Reya [14] in which the pQCD calculations have no predictive power regarding the normalization of $F_2^p$ but retain, as for a proton target, the $Q^2$ dependence of the evolution equations.

The above philosophy is very appropriate for our program where we distinguish between the hard pQCD mode and the non-perturbative QCD (npQCD) soft mode of the gluon fields by introducing [1,15], two separation parameters ($M_{0T}^2$ and $M_{0L}^2$) in which we match the long and short distance components of the transverse (T) and longitudinal (L) contributions to the total $\gamma^*\gamma^*$ cross section. Our ideology is close to the semiclassical gluon field approach developed in [16]. This approach allows one to find a relation between scattering amplitudes and the property of the QCD vacuum based on the model of the stochastic vacuum (MSV) [17]. Whereas the MSV is guided by the assumption of a microscopic structure of the QCD vacuum, our model is phenomenologically based on the additive quark model (AQM) [18]. The MSV has been combined [11] with the two Pomeron model [19]. In the two Pomeron model the hard Pomeron is a fixed $J$-pole whose $Q^2$ dependence is determined by fitting to the data. In a pQCD calculation of the hard Pomeron, one has an effective $J$-pole whose dependence on $x$ and $Q^2$ is determined by $xG(x,Q^2)$. A short review of the various approaches to $\gamma^*\gamma^*$ reactions at high energies, stressing the need for a simultaneous determination of both the soft and the hard contributing components, has just appeared [20].

The plan of our paper is as follows: In Sect. 2 we review the generalization of the ideas presented in [1] and outline the expansion of this model for the $\gamma^*\gamma^*$ cross section. In Sect. 3 we derive the complete set of formulæ for the total cross section components. We present the details of our numerical calculations in Sect. 4 and compare our results with the high energy experimental data available to date. Our conclusions are summarized in Sect. 5.

2 Review of the approach

Our approach follows from the ideas presented in [21]. This was first suggested in [15] and successfully applied in [1]. According to Gribov’s general approach [2], the interaction of a virtual photon, in any QCD description, can be interpreted as a two stage process. The first stage is the fluctuation of the photon into a hadronic system, and in the next stage the hadronic system interacts with the “target”, which in our case is another hadronic system from a different parent photon (see Fig. 1). These two processes are time ordered and can be treated independently. The vertex function $\Gamma(M^2)$ of the photon fluctuation into a $q\bar{q}$ pair of mass $M$ is given by the experimental value of the ratio

$$\Gamma(M^2) = R(M^2) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}.$$ (1)

The complete process of two virtual photons fluctuating into two quark–antiquark pairs which then interact with each other can be expressed by the following dispersion relation:

$$\sigma(\gamma^*\gamma^*) = \frac{(\alpha_{\text{em}}/3\pi)^2}{\Gamma(M^2)} \int dM_1^2 dM_2^2 dM_1'^2 dM_2'^2 \times \frac{\Gamma(M_1^2)}{(1 + M_1^2)} \frac{\Gamma(M_2^2)}{(1 + M_2^2)}$$

$$\times \sigma(M_1^2, M_1'^2; M_2^2, M_2'^2; s) \frac{\Gamma(M^2)}{(Q_1^2 + M_1^2)}$$

$$\times \frac{\Gamma(M^2)}{(Q_2^2 + M_2^2)}.$$ (2)

where $\sigma(M_1^2, M_1'^2; M_2^2, M_2'^2; s)$ is the cross section of the interaction between two hadronic systems with masses $M_1$ and $M_2$ before the interaction and $M_1'$ and $M_2'$ after the interaction.

We introduce a separation parameter in the mass integrations, which may be different for a longitudinal and transverse polarized virtual photon ($M_{0L}$ and $M_{0T}$, respectively). For masses below this parameter, the process is soft, long range, and hence one cannot describe the produced hadron state as a $q\bar{q}$ pair. For masses above the separation parameter, the distances between the quark and antiquark are short, and we shall derive the formulæ for masses above this parameter. For masses above the separation parameter, the distances between the quark and antiquark are short, and we shall derive the formulæ for masses above this parameter. For masses above this parameter, the distances between the quark and antiquark are short, and we shall derive the formulæ for masses above this parameter. For masses above this parameter, the distances between the quark and antiquark are short, and we shall derive the formulæ for masses above this parameter.

The calculation of the two photon total cross section, according to our approach, is derived following the same concepts of the $\sigma(\gamma^*p)$ calculations. Each of the two photons can be soft or hard, and we shall derive the formulæ on this basis. Without loss of generality, we shall consider the case in which one photon (say, the upper one) is “harder” than the other, hence there are three sectors of the calculation:

(a) “Hard–hard” when both photons are hard. We treat the interaction between the two $q\bar{q}$ pairs in pQCD, calculating all the diagrams in which the upper $q\bar{q}$ pair are harder than the gluons in the ladder, and the gluons in the ladder are harder than the lower pair $k_1^2 > k_2^2 > k_2^2 > k_2^2$ (see Fig. 2). The cross section of the interaction in the hard–hard sector can be expressed through $xG_s$, the dis-