Predictions for $\bar{\nu}\nu\gamma$ production at LEP*

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Abstract. We study predictions for the reaction $e^+e^- \to \bar{\nu}\nu(n\gamma)$. The complete one-loop corrections are taken into account and higher order contributions, in particular those for the observed real photons, are added whenever necessary. The event generator KK MC, a general-purpose Monte Carlo generator for the process $e^+e^- \to ff\gamma$ based on the method of exclusive exponentiation, is used as the environment. We extend its applicability to the process $e^+e^- \to \bar{\nu}\nu(n\gamma)$, $l = e, \mu, \tau$, where the observation of at least one single $\gamma$ is required. The exponentiation is implemented in much the same way as for the $s$-channel process alone. In particular, all photonic effects present in the case of $W$ exchange, which cannot be included in the $s$-channel exponentiation scheme, are calculated to a finite order only. The real hard photon matrix element is calculated up to $O(\alpha^2)$. Leading logarithmic contributions of the two-loop corrections and one-loop photonic corrections accompanying real single-photon emission are included. The electroweak corrections are calculated with the DIZET library of the ZFITTER package. Numerical tests and predictions for typical observables are presented.

1 Introduction

For the final LEP2 data analysis the total cross-section for the process $e^+e^- \to \bar{\nu}\nu\gamma$ will have to be calculated with a precision of 0.5%–1%, and arbitrary differential distributions of observable photons also have to be calculated with similar precision [1]. In the future, for a high luminosity linear electron collider like TESLA, the precision requirements will be even more demanding. These requirements may be fulfilled by transferring our expertise from the case of charged lepton production to that of neutrino production, in particular the more involved case of $\bar{\nu}_e\nu_e$ with $t$-channel exchanges. The present paper marks an important step in this direction. It contains the necessary extensions of the Monte Carlo program KK MC of [2], originally written for $e^+e^- \to ff$, $f = \mu, \tau, u, d, c, s, b$, to the neutrino case.

In the neutrino pair production process

$$e^+e^- \to \bar{\nu}\nu\gamma,$$  (1.1)

one is interested in observables where at least one high-$p_T$ photon is observed; neutrinos obviously escape detection. From a methodological point of view, however, it is convenient to consider

$$e^+e^- \to \bar{\nu}\nu$$  (1.2)

as a (non-observable) Born process, and to incorporate (observable) radiative corrections into it, in particular the real photon emissions, which provide the detectable signature. A convenient method of exponentiation is discussed in this framework. In order to achieve the 0.5% precision level for the $\bar{\nu}\nu\gamma$ final state, the leading-logarithmic (LL) corrections have to be calculated up to two or three loops for the virtual corrections and up to two or even three hard photons for multiple bremsstrahlung. Mixed real–virtual terms such as the loop corrections to real photon emission have to be calculated as well$^1$. Needless to say that a sufficiently precise integration over the multiphoton phase space within the detector acceptance is also necessary. The Monte Carlo (MC) event generator approach is the only practical solution.

As in the case of any other two-fermion final state in $e^+e^-$ scattering, it is possible to define certain computational building blocks. Our case does not require the complete two-loop effects and we can separate the calculation into two parts:

$^1$ The genuine weak one-loop corrections are sufficient.
(i) QED: interaction of photons with fermions as well as $W W \gamma$ and $W W \gamma \gamma$ interactions; and
(ii) the rest: non-photonic weak and QCD corrections.

The type (ii) corrections can be hidden in a few effective coupling constants.

The Monte Carlo method is used for the numerical integration over the Lorentz-invariant phase space, as usual. The Monte Carlo event generator KK MC is documented in [2]. For a detailed description of its matrix elements for the $e^+e^- \rightarrow f f (\gamma \gamma)$ processes we refer the reader to [3,4].

In Sect. 2 we discuss the implementation of the electroweak corrections. The package ZFITTER [5,6] is used for this purpose. Basic numerical tests of the code in the absence of photonic effects are described.

In Sect. 3 we introduce the photonic matrix elements. We start from the simplest cases of $\nu e$ and $\nu \nu$ production. We then explain the extension of the matrix elements used in the CEEX exponentiation of [2–4], to the case of $e^+e^- \rightarrow \bar{\nu}_e \nu e \gamma$. The modifications are due to the presence of $t$-channel $W$ exchange. We start with the general description of our approximation, and later present the single-photon tree-level amplitude. In particular, we explain how single bremsstrahlung amplitudes are used as a building block in the multiple-photon amplitudes. Finally we briefly explain the calculation (or construction) of our amplitudes for the different higher order cases: one virtual and one real photon, and two real photons.

In Sect. 4, predictions of KK and KORALZ [7,8] are given for selected observables of the recent LEP MC workshop [1], including results that are used for the final estimate of the theoretical and technical errors of our new calculation.

Section 5 concludes this paper with a statement on the precision of our theoretical predictions for the $\bar{\nu} \nu \gamma$ process, as compared with the precision targets requested by LEP experiments [1].

### 2 The effective Born approximation for $e^+ e^- \rightarrow \bar{\nu} \nu$

Similarly to the case of pure $s$-channel two-fermion processes, the electroweak one-loop corrections can be incorporated via effective coupling constants of the $Z$ and $W$ to fermions. Let us define here the complete electroweak processes, the electroweak one-loop corrections can be incorporated via effective coupling constants of the $Z$ and $W$ to fermions. Let us define here the complete electroweak one-loop corrected effective Born cross-section for neutrino pair production. This is by construction a gauge-invariant quantity. It includes $s$-channel $Z$ exchange for all three neutrino species, while for $\nu e$ pair production it also includes $t$-channel $W$ exchange:

$$
\frac{d\sigma}{d \cos \theta} = \sum_{i=e,\mu,\tau} \frac{d\sigma(e^+e^- \rightarrow \bar{\nu}_i \nu_i)}{d \cos \theta} = 3\sigma_s + \sigma_{st} + \sigma_t.
$$

The improved Born cross-section originates from a neutral-current matrix element $M_Z$ [5,9],

$$
M_Z = \frac{G_\mu}{2\sqrt{2}} Re\chi Z(s) [\bar{u}_e \gamma_{\mu} (\bar{v}_e + \gamma_5) u_e] 
\times [\bar{u}_e \gamma_{\mu} (1 + \gamma_5) u_e],
$$

and, for $\bar{\nu}_e \nu_e$ production only, additionally from a charged current matrix element $M_W$ [10]:

$$
M_W = \frac{G_\mu}{\sqrt{2}} Re\chi W(t) [\bar{u}_e \gamma_{\mu} (1 + \gamma_5) u_e]
\times [\bar{u}_e \gamma_{\mu} (1 + \gamma_5) u_e].
$$

We use here the notations $a_e = a_\nu = 1$, $Q_\nu = -1$, $s_W^2 = 1 - M_W^2/M_Z^2$, and have only three form factors $\kappa_e$, $\rho^{Z\gamma}_e$, and $\rho^{e\gamma}_e$:

$$
\bar{v}_e = -4Q_e s_W^2 \kappa_e.
$$

In the Born approximation, it is $\rho = \kappa = 1$. The kinematical invariants are used in the approximation $m_e = 0$:

$$
t = -\frac{s}{2} (1 - \cos \theta),
$$

$$
u = -s - t = -\frac{s}{2} (1 + \cos \theta).
$$

We also use

$$
\chi_B(s) = \frac{M_B^2}{-s + M_B^2(s)},
$$

$$
M_B^2(s) = M_B^2 - i M_B \Gamma_B(s) \theta(s),
$$

and the width in the $s$-channel may be chosen constant or $s$-dependent. The resulting cross-section contributions are

$$
\sigma_s = \frac{sG_\mu^2}{128\pi} |\chi Z(s)\rho^{Z\gamma}_e|^2 
\times \left( (1 + \cos^2 \theta)(1 + |v_e|^2) + 4 \cos \theta \text{Re} v_e \right),
$$

$$
\sigma_{st} = \frac{sG_\mu^2}{32\pi} \text{Re} \left\{ \chi Z(s) \chi W(t) \rho^{Z\gamma}_e \rho^{e\gamma}_e \rho_{e\nu} W^* 
\times (1 + \cos \theta)^2 (1 + v_e) \right\},
$$

$$
\sigma_t = \frac{sG_\mu^2}{16\pi} |\chi W(t)\rho^{e\gamma}_e|^2 (1 + \cos \theta)^2.
$$

The weak neutral form factors $\kappa_e$ and $\rho^{Z\gamma}_e$ are discussed in detail in [5,11] and in references therein. For the latest comparisons and applications at LEP1, see also [12], and at LEP2, see [1]. There is one modification with respect to earlier applications, which has to be clarified here. Even though the complete virtual corrections are not infrared-finite (because of photonic diagrams), we prefer

$^3$ These form factors are calculated in the library DIZET as variables XROK(2) and XROK(1) by calling subroutine ROKANC($u,-s,t$) for $s > 0$ and $t,u < 0$