The hard bremsstrahlung correction to $e^+e^- \rightarrow 4f$
with finite fermion masses: results for $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu^*$

F. Jegerlehner¹, K. Kołodziej²

1 Deutsches Elektronen-Synchrotron DESY, Platanenallee 6, 15708 Zeuthen, Germany
2 Institute of Physics, University of Silesia, ul. Uniwersytecka 4, 40007 Katowice, Poland

Received: 5 July 1999 / Published online: 16 November 1999

Abstract. An improved efficient method of calculating the hard bremsstrahlung correction to $e^+e^- \rightarrow 4f$ for non-zero fermion masses is presented. The non-vanishing fermion masses allow us to perform the phase space integrations to the very collinear limit. We therefore can calculate cross sections independent of angular cuts. Such calculations are important for background studies. Results are presented for the total and some differential cross sections for $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ and the corresponding hard bremsstrahlung process. The latter is of particular interest for a detailed investigation of the effects of final state radiation. In principle, the process $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ is also interesting since it helps to set bounds on possible anomalous triple and quartic gauge boson couplings involving photons. The size of mass effects is illustrated by comparing the final states $u\bar{d}\mu^-\bar{\nu}_\mu(\gamma)$, $c\bar{s}\mu^-\bar{\nu}_\mu(\gamma)$ and $u\bar{d}\tau^-\bar{\nu}_\tau(\gamma)$.

1 Introduction

Precision measurements of properties of the intermediate gauge bosons $Z$ and $W$ have deepened our understanding of the electroweak interactions and consolidated the validity of the electroweak Standard Model (SM) considerably in the past decade. After the convincing success of LEP1 and SLD experiments in pinning down the properties of the $Z$ resonance we expect further advances in measurements of the properties of the $W$ boson which are not yet known with comparable precision. As the SM very precisely predicts the mass and the width of the $W$ a high accuracy determination of these parameters is of crucial importance, because they allow us to improve indirect bounds on the Higgs mass or on new physics beyond the SM. The precision of ongoing measurements, single $W$ production at the hadron collider TEVATRON, and $W$ pair production at LEP2 are limited by statistics and/or by lack of detailed theoretical understanding.

The proper analysis of $W^{\pm}$ pair production at LEP2 and later at future high energy $e^+e^-$ linear colliders requires the accurate knowledge of the SM predictions including all relevant radiative corrections. What we need is a detailed understanding of the production and subsequent decay of $W$ pairs, including the background processes and photon radiation: $e^+e^- \rightarrow 4f$, $4f\gamma$, $4f\gamma\gamma$, ..., where $4f$ denotes a possible four fermion final state. The lowest order theoretical results for all the possible four fermion final states have been already implemented in several Monte Carlo event generators and semi-analytic programs, which have been thoroughly compared in [1]. Most of the programs include some classes of radiative corrections such as the initial and final state radiation, Coulomb corrections, running of the fine structure constant, etc. While presently available $e^+e^- \rightarrow 4f$, $4f\gamma$ matrix elements are precise enough for the analysis of LEP2 data [2], at future linear colliders, a much better knowledge of the radiative corrections will be necessary because of the high statistics expected at these accelerators and because radiative corrections get more significant at higher energies.

The complete one-loop electroweak radiative corrections to the on-shell $W^{\pm}$ pair production including soft bremsstrahlung were calculated in [3]. The hard bremsstrahlung process $e^+e^- \rightarrow W^+W^-\gamma$ was included in [4] and [5]. For the process $e^+e^- \rightarrow W^+W^- \rightarrow 4f$ of actual interest to the experiments only partial results are available. We refer to [6] for a recent review of the status of precision calculations for this case.

Sufficiently above the $W^{\pm}$ pair production threshold, for most of the present applications, it seems to be sufficient to take into account corrections to the double-resonant diagrams only, i.e., $e^+e^- \rightarrow 4f$ via virtual $W^+W^-$ intermediate states. The validity of this approximation has to be controlled by more complete calculations, however. From a theoretical point of view it is certainly necessary to evaluate the complete $O(\alpha)$ radiative corrections for the different channels of the $2 \rightarrow 4$ fermion reactions. However, despite of the fact that some progress in calculating the complete virtual one-loop electroweak radiative corrections to $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ have been reported in [7], the
final result of such a calculation is still missing. Concerning the real photon emission, the situation looks much better. The hard bremsstrahlung for four fermion reactions mediated by two resonant $W$ bosons was calculated in [8]. A similar calculation, extended by an inclusion of collinear effects, was presented as a package WWF [9]. The complete lowest order result for $e^+e^- \to e^+e^- ud\gamma$ was presented in [10] and calculations of $e^+e^- \to 4f\gamma$ for an arbitrary final state were reported in [11]. Results on bremsstrahlung for purely leptonic reactions have been published in [6] and most recently predictions for all processes $e^+e^- \to 4f\gamma$ with massless fermions have been presented in [12].

At a future linear collider, the proper treatment of the collinear photons will be crucial and it requires to take into account the fermion masses appropriately. Therefore, in the present paper, we propose an efficient method of calculating the hard photon bremsstrahlung for four fermion production in $e^+e^- \to 4f\gamma$ annihilation without neglecting the fermion masses. The phase space integration can therefore be performed to the very collinear limit. This allows for calculating cross sections independent of angular cuts and estimating background contributions coming from undetected hard photons. We present results for the total and a few differential cross sections for the channel $e^+e^- \to ud\mu^-\bar{\nu}_\mu$, and the corresponding bremsstrahlung process. The latter is particularly suited for a detailed investigation of effects related to final state photon emission, since the muons appear well separated from photons in the detectors. In particular, it seems to be interesting to study the influence of final state radiation on the $W$ mass measurement via this channel. Having the final state photon resolution in $e^+e^- \to ud\mu^-\bar{\nu}_\mu\gamma$ could also make it possible to investigate the quartic $\gamma VWW$ couplings ($V = \gamma, Z$), which are absent on the Born level of $4f$ production. Of course, besides the new quartic couplings there are additional triple $\gamma VWW$ vertices as well. In the soft photon limit, we can perform the integration over the soft photon phase space analytically and demonstrate the cut-off independence of the combined soft and hard photon bremsstrahlung cross section. We finally will illustrate the importance of mass effects by comparing the channels where $ud$ is replaced by $cs$. Similarly, we may replace the $\mu$ by a $\tau$ lepton.

## 2 Method of calculation

In this section, we present a method for calculating the matrix elements of a two-fermion to four fermion reaction and an associated bremsstrahlung photon. The method is an extension of the helicity amplitude method introduced in [4] to final states of arbitrary spin.

As in [4], we use the Weyl representation for fermions where the Dirac matrices $\gamma^\mu, \mu = 0, 1, 2, 3,$ are given in terms of the unit $2 \times 2$ matrix $I$ and Pauli matrices $\sigma_i, i = 1, 2, 3,$ by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \sigma^\mu & 0 \end{pmatrix}, \quad (1)$$

with $\sigma^\mu_i = (I, \pm \sigma_i)$. In representation (1), the matrix $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and the chiral projectors $P_{\pm} = (1 \pm \gamma_5)/2$ read

$$\gamma_5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}, \quad P_- = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \quad P_+ = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}. \quad (2)$$

A contraction of any four-vector $a^\mu$ with the $\gamma^\mu$ matrices of (1) has the form

$$a^\mu \gamma_5 = \begin{pmatrix} 0 & a^\mu \sigma^+ \sigma^- \\ a^\mu \sigma^+ \sigma^- & 0 \end{pmatrix} = \begin{pmatrix} 0 & a^+ \\ a^- & 0 \end{pmatrix}. \quad (3)$$

The $2 \times 2$ matrices $a^\pm$ can be expressed in terms of the components of the four-vector $a^\mu$ by

$$a^+ = \begin{pmatrix} 0 & -a^0 + a^3 \\ -a^1 - ia^2 & a^0 - a^3 \end{pmatrix}, \quad a^- = \begin{pmatrix} 0 & a^0 + a^3 \\ a^1 + ia^2 & a^0 - a^3 \end{pmatrix}. \quad (4)$$

In representation (1), the helicity spinors for a particle, $u(p, \lambda)$, and an antiparticle, $\bar{v}(p, \lambda)$, of four-momentum $(E, p)$ and helicity $\lambda = \pm 1/2$ are given by

$$u(p, \lambda) = \begin{pmatrix} \sqrt{E - \lambda |p|} \chi(p, \lambda) \\ \sqrt{E + \lambda |p|} \chi(p, \lambda) \end{pmatrix},$$

$$\bar{v}(p, \lambda) = \begin{pmatrix} -\lambda \sqrt{E + |p|} \chi(p, -\lambda) \\ \lambda \sqrt{E - |p|} \chi(p, -\lambda) \end{pmatrix}, \quad (5)$$

and the helicity eigenstates $\chi(p, \lambda)$ can be expressed in terms of the spherical angles $\theta$ and $\phi$ of the momentum vector $p$ as\footnote{Note that our phase convention differs from the one chosen in [4].}

$$\chi(p, +1) = \begin{pmatrix} \cos \theta/2 \\ e^{i\phi} \sin \theta/2 \end{pmatrix}, \quad (6)$$

For simplicity we use real polarization vectors which are defined again in terms of $\theta$ and $\phi$

$$e^\mu(p, 1) = (0, \cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta), \quad (7)$$

$$e^\mu(p, 2) = (0, -\sin \phi, \cos \phi, 0), \quad (7)$$

$$e^\mu(p, 3) = \gamma (\beta, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta), \quad (8)$$

where the longitudinal polarization component of (8) is defined exclusively for a massive vector particle of energy $m\gamma$ and momentum $m\gamma \beta$. We could use complex polarization vectors in the helicity basis as well, if we were interested in definite helicity polarizations.

A polarized matrix element is calculated for a given set of external particle momenta in a fixed reference frame,