Radiative effects in processes of diffractive vector meson electroproduction

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Abstract. The electromagnetic radiative correction to the cross section of the vector meson electroproduction is calculated. Explicit covariant formulae for the observed cross section are obtained. The dependence of the radiative correction on the experimental resolution and on the inelasticity cut is discussed. The FORTRAN code DIFFRAD, based on both exact (ultrarelativistic) and approximate sets of the formulae for the radiative correction to the cross section, is presented. Detailed numerical analysis for kinematical conditions of the recent experiments on the diffractive electroproduction of vector mesons is given.

1 Introduction

The measurement of the cross section of the exclusive vector meson electroproduction can provide information on the hadronic component of the photon and on the nature of diffraction. Over several years, the diffractive production of the vector meson has been the subject of muon production [1–3] and electroproduction [4–6] experiments. Data analysis of these experiments is affected considerably by QED radiative effects. In practice, the radiative corrections (RC) to the processes of electroproduction are taken into account by using codes originally developed for the inclusive case (see [7], for example).

The purpose of this paper is to calculate the electromagnetic correction to experimentally observed cross sections for the kinematics of fixed target and collider experiments directly. The Feynman diagrams necessary to calculate RC are presented on Fig. 1.

In order to calculate exactly the QED RC to the cross section of vector meson production, the method offered in [8] is used. By exact formulae we mean the expressions for the lowest-order RC obtained without any approximations but ultrarelativistically: the lepton mass \( m \) is considered to be small. In Sect. 2 the kinematics of the radiative and non-radiative processes and exact formulae for the lowest-order RC are obtained. In Sect. 3 the analytical results are visualized by the construction of the approximate formulae for cases interesting in practice. The numerical results are given in Sect. 4. A brief discussion and conclusions are given in Sect. 5.

2 Exact formulae for the lowest-order correction

Seven kinematical variables are necessary to describe the radiative process of diffractive vector meson production (Fig. 1b,c). Four of them are the same as for the non-radiative case: the usual scaling variables \( x \) and \( y \), the negative square of momentum transferred from the virtual photon to the proton \( t = (q - p_h)^2 \) and the angle \( \phi_h \) between the scattering \( (k_1, k_2) \) and production \( (q, p_h) \) planes in the laboratory frame. The squared virtual photon momentum \( Q^2 = -(k_1 - k_2)^2 \) and the invariant mass of the initial proton and the virtual photon \( W^2 = (p + q)^2 \) are often used instead of \( x \) and \( y \). The kinematics of a real photon is described by three additional variables [9,

\[
\begin{align*}
&k_1 & &q & &k_2 \\
p & &p_h & &p' \\
&\text{a)} & &\text{b)} & &\text{c)}
\end{align*}
\]

Fig. 1. Feynman diagrams contributing to the Born and the next-order cross sections. The letters denote the four-momenta of corresponding particles.
The way to calculate integrals like (6) has been offered in [8], where it was shown that the squared invariant mass of the system of unobserved particles, \( \tau = kq/kp \) and the angle \( \phi_k \) between planes \((k_k, k_2)\) and \((q, k)\).

We consider the RC to three- and four-dimensional cross sections \( \sigma = d^4\sigma/dx dy dt d\phi_h \) and \( \sigma = d^3\sigma/dx dy dt \). They are related as
\[
\bar{\sigma} = \int_{0}^{2\pi} d\phi_h \sigma. \tag{1}
\]
The four differential Born cross section can be presented in the form
\[
\sigma_0 = \frac{\alpha}{4\pi^2xy} \left( y^2 \sigma_T + 2(1 - y - \frac{1}{4}y^2) \right) (\sigma_T + \sigma_L), \tag{2}
\]
where \( \sigma_T \) and \( \sigma_L \) are differential cross sections of the photoproduction, \( \gamma^* = Q^2/\nu^2 \) and \( \nu \) is the virtual photon energy.

The differential cross section of the radiative process has the following form:
\[
\sigma_R \propto \left| M_h + M_c \right|^2 \frac{d^4k}{2k_0} \delta ((A - k)^2 - M^2), \tag{3}
\]
where \( A = p + q - p_h \) and \( M_{h,c} \) are matrix elements of the processes given in Fig. 1a, b. In order to extract the infrared divergence and form it into a separate term we follow [8] and use the identity
\[
\sigma_R = \sigma_R - \sigma_R + \sigma_{IR} = \sigma_F + \sigma_{IR}. \tag{4}
\]
where \( \sigma_R \) is finite for \( k \to 0 \) and \( \sigma_{IR} \) is the infrared divergent part
\[
\sigma_{IR} = \frac{\alpha}{\pi} \delta R_0 \sigma_0 = \frac{\alpha}{\pi} (\delta_S + \delta_H) \sigma_0. \tag{5}
\]
The quantities \( \delta_S \) and \( \delta_H \) appear after splitting the integration region over \( \nu \) by the infinitesimal parameter \( \bar{\nu} \):
\[
\delta_S = \frac{1}{\pi} \int_{0}^{\bar{\nu}} d\nu \int \frac{d\nu' d^3k}{(2\mu\nu'^n - 3k_0)} F_{IR} \delta ((A - k)^2 - M^2), \tag{6}
\]
\[
\delta_H = \frac{1}{\pi} \int_{0}^{\bar{\nu}} d\nu \int d^3k k_0 F_{IR} \delta ((A - k)^2 - M^2), \tag{7}
\]
where \( \mu \) is an arbitrary parameter of the mass dimension, \( \nu_m \) is a maximal inelasticity and
\[
F_{IR} = \frac{m^2}{(2kk)^2} + \frac{m^2}{(2kk)^2} - \frac{Q^2 + 2m^2}{(2kk)(2kk)}. \tag{8}
\]
The way to calculate integrals like (6) has been offered in [8] (see also the review [9]). In our case we have
\[
\delta_S = 2 \left( P_{IR} \right) \left( 1 + \nu_m \right) (l_m - 1) \left( \frac{\nu}{\nu_M} \right) S_{IR} + \log \frac{S'X'}{S'X}, \tag{9}
\]
\[
\delta_H = 2(l_m - 1) \log \frac{\nu_m}{\bar{\nu}}, \tag{10}
\]
where \( l_m = \log(Q^2/m^2) \). The quantities \( S' = 2Ak_1 = S - Q^2 - V_1 \) and \( X' = 2Ak_2 = X + Q^2 - V_2 \) are calculated by using \( V_{1,2} = 2(a_{1,2} + b \cos \phi_h) \), where
\[
a_1 = \frac{1}{2\lambda_q} (Q^2 S_{p;S} - (SS_x + 2m^2Q^2)t_q), \tag{11}
\]
\[
a_2 = \frac{1}{2\lambda_q} (Q^2 S_{p;S} - (XS_x - 2m^2Q^2)t_q), \tag{12}
\]
\[
b = \frac{1}{\lambda_q} (Q^2 S_{p;S} - (SS_x - 2m^2Q^2)t_q - 2m^2\lambda_q)^{1/2} \times (S'XQ^2 - 2M^2Q^2 - m^4\lambda_q)^{1/2}. \tag{13}
\]
The invariants are defined as
\[
S = 2k_kp, \quad X = 2k_kp = (1 - y)S, \quad Q = S_{xy}, \quad \lambda_q = S^2 + 4M^2Q^2. \tag{14}
\]

The infrared terms \( P_{IR} \), parameters \( \mu \) and \( \bar{\nu} \) and the squared logarithms containing the mass singularity \( \nu_m \) are completely canceled in the sum of \( \delta_{IR} \), with \( \delta_{IR} \) coming from a contribution of the vertex function (Fig. 1d):
\[
\delta_{IR} = -2 \left( P_{IR} + \log \frac{\nu}{\mu} \right) (l_m - 1) \left( \frac{1}{2} l_m^2 + \frac{3}{2} l_m - 2 + \frac{\pi^2}{6} \right). \tag{15}
\]

For this sum we have
\[
\frac{\alpha}{\pi} (\delta_S + \delta_H + \delta_{IR}) = \delta_{int} + \delta_{VR}, \tag{16}
\]
where
\[
\delta_{VR} = \frac{\alpha}{\pi} \left( \frac{3}{2} l_m^2 - 2 - \frac{1}{2} \log \frac{Q^2}{S'} \right) + \log \left( 1 - \frac{Q^2}{S'} \right) \left( \frac{S'X'}{S'X} \right) \left( \frac{\mu}{\nu_M} \right), \tag{17}
\]
\[
\delta_{int} = \frac{\alpha}{\pi} (l_m - 1) \log \frac{\nu_m}{\bar{\nu}}. \tag{18}
\]
Here we used the ultrarelativistic expression for \( S_\phi \) calculated in [12]. The higher-order corrections can be partially taken into account by using a special procedure of exponentialization of the multiple soft photon radiation. There is an uncertainty: what part of \( \delta_{VR} \) has to be exponentiated? Within the considered approach [12] (1 + \( \delta_{int} \)) is replaced by \( \exp \delta_{int} \).

For the observed cross section of the vector meson electroproduction we obtain
\[
\sigma_{obs} = \sigma_0 \delta_{int} (1 + \delta_{VR} + \delta_{vac}) + \sigma_F. \tag{19}
\]

The correction \( \delta_{vac} \) comes from the effects of vacuum polarization by leptons and hadrons (Fig. 1e). The explicit QED formulae for the first one can be found in [9]. The hadronic contribution is given by a fit coming from the data on \( e^+ e^- \to \text{hadrons} \) [13].