Environment-induced nonclassical behaviour

Paulina Marian\textsuperscript{1,\textit{a}} and Tudor A. Marian\textsuperscript{2}

\textsuperscript{1} Laboratory of Physics, Department of Chemistry, University of Bucharest, Boulevard Carol I 13, 70346 Bucharest, Romania
\textsuperscript{2} Department of Physics, University of Bucharest, P.O. Box MG-11, 76000 Bucharest-Măgurele, Romania

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Abstract. We analyze the transient nonclassical behaviour of a single-mode field whose interaction with an environment is governed by the quantum optical master equation. Our analytic method makes use of the generalized characteristic function of the field state. First, we find a time at which all squeezing effects disappear by decoherence regardless of the initial state of the mode. In the case of an input even coherent state, an unusual modification of higher-order squeezing at low values of thermal mean occupancy transferred to the field is found and discussed. For the same initial state, we also perform a comprehensive analysis of the mixing process during the interaction with the reservoir. We prove that a maximum in the evolution of the 2-entropy of the attenuated mode exists on condition that its initial mean photon number exceeds the mean occupancy of the reservoir. This transient mixing enhancement can be considered as a quantum effect of the initial state on the mode damping.

PACS. 42.50.Dv Nonclassical field states; squeezed, antibunched, and sub-Poissonian states; operational definitions of the phase of the field; phase measurements – 03.65.Bz Foundations, theory of measurement, miscellaneous theories (including Aharonov-Bohm effect, Bell inequalities, Berry’s phase)

1 Introduction

Interaction of a quantum system, such as a single-mode radiation field, with an environment has been intensively studied in connection with the emergency of classicality for the system [1–3]. In reference [1], the master equation of the quantum Brownian motion in the high-temperature limit has been used to get a predictability sieve for the effectiveness of quantum decoherence. In references [2,3], effects of the environment on a harmonic oscillator have been investigated in the framework of the Lindblad equation formalism [4]. Special attention is paid to the case of an interaction which is linear in position and momentum [5]. The works [1–3] have exploited the production of reduced linear entropy as an instrument for determining maximal states of the system. Examination of the von Neumann entropy production deduced from the master equation of quantum Brownian motion [6] reveals that information about the system diminishes considerably at short times. Simultaneously, dramatic changes of nonclassical properties of the field mode could happen. These properties generally soften owing to the dissipative interaction [7–9]. However, recent works [10–12] report that, in some cases, environment enhances nonclassical features. Specifically, a single-mode field in a superposition of coherent states weakly coupled to a heat bath at zero [10] or very low temperature [11] has been studied. It has been found that fourth-order squeezing could be transiently created due to the interaction with the reservoir described by the quantum optical master equation [13]. Note that, according to references [2–5], this master equation is precisely of the type introduced by Lindblad.

Use will be made in the present paper of the quantum optical master equation in order to elucidate two problems:

(i) occurrence of higher-order squeezing for a nonclassical single-mode radiation field coupled to a low-temperature heat bath. As an illustration, the case of an initial even coherent state (ECS) [14] is explicitly treated;
(ii) evolution of the mixing process undergone by an input ECS.

The paper is organized as follows. In Section 2 we first discuss the solution of the master equation in terms of the characteristic function (CF) of the damped mode. Then, we give general analytic formulae describing squeezing to any even order $N$. They are applied in Section 3 to the interesting case of an ECS. We also point out the loss of nonclassicality by means of the $P$-representation of the density operator. The 2-entropy of a damped ECS is evaluated in Section 4 making use of its CF. Clearly, the 2-entropy turned out to be an efficient tool to analyze the evolution of the mixing produced by the environment. We distinguish a classical behaviour of this mixing from a nonclassical one by comparing the initial mean photon number of the field with the mean occupancy of the reservoir.

\textit{a} e-mail: pmarian@math.math.unibuc.ro
2 Evolution of higher-order squeezing

We denote by \( a \) and \( a^\dagger \) the amplitude operators of the field mode. The quantum optical master equation in the interaction picture is [13]

\[
\frac{\partial \rho}{\partial t} = \gamma (\overline{n}_R + 1)(2a^\dagger a\rho - a^\dagger a\rho - \rho a^\dagger a) + \gamma \overline{n}_R (2a^\dagger \rho a - aa^\dagger \rho - \rho aa^\dagger).
\]  

(2.1)

In equation (2.1), \( \rho \) is the reduced density operator of the field, \( \gamma \) is the coupling constant between field and bath, and \( \overline{n}_R \) stands for the mean occupancy of the reservoir.

The usefulness of the normally ordered generalized characteristic function (GCF) recently introduced in our paper [12] as

\[
\chi_N(\lambda, \lambda') := \langle \exp (\lambda a^\dagger) \exp (\lambda' a) \rangle
\]  

(2.2)

is proved once more when applying it to equation (2.1). By use of well-known methods [7,13], we convert the master equation (2.1) into a first-order partial differential equation for the GCF (2.2):

\[
\frac{\partial \chi_N}{\partial t} = \gamma \overline{n}_R \chi_N - \frac{\gamma}{2} - i\omega \chi_N - \frac{\gamma}{2} + i\omega \chi_N.
\]  

(2.3)

The solution of this equation found by the characteristic-curve method [15] depends on its initial form \( \chi_N(\lambda, \lambda', 0) \) as

\[
\chi_N(\lambda, \lambda', t) = \chi_N(\lambda e^{-(\gamma/2 - i\omega)t}, \lambda' e^{-(\gamma/2 + i\omega)t}, 0) \times \exp \left[ \overline{n}_R (1 - e^{-\gamma t}) \lambda \lambda' \right].
\]  

(2.4)

Note that the GCF (2.2) is picture independent. When \( \lambda' = -\lambda', \) equation (2.2) gives the usual normally ordered CF \( \chi_N(\lambda) \) introduced by Glauber [16]. By employing this function, one readily finds the expectation values

\[
\langle (a^\dagger)^m a^n \rangle = (-1)^m \left[ \frac{\partial^{m+n}}{\partial \lambda^m \partial \lambda'^n} \chi_N(\lambda) \right]_{\lambda = 0},
\]  

(2.5)

which are necessary when one has to examine the statistical properties of the field state. For instance, the mean photon number \( \langle l \rangle = m = 1 \) in Eq. (2.5)) is obtained from the normally ordered CF (2.4) as

\[
\overline{n}(t) = \overline{n}(0) e^{-\gamma t} + \overline{n}_T(t),
\]  

(2.6)

where

\[
\overline{n}_T(t) := \overline{n}_R [1 - \exp (-\gamma t)]
\]  

(2.7)

is the thermal mean occupancy in the field mode at time \( t \). Note also that, if existing as a well-behaved function, the Fourier transform of the normally ordered CF is the Glauber-Sudarshan \( P \)-representation [17]

\[
P(\beta) = \frac{1}{\pi} \int d^2 \lambda \exp (\beta \lambda^* - \beta^* \lambda) \chi_N(\lambda).
\]  

(2.8)

The density operator is fully determined by the symmetrically ordered CF,

\[
\chi(\lambda) = \exp (-|\lambda|^2/2) \chi_N(\lambda),
\]  

via the Weyl expansion [18]

\[
\rho = \frac{1}{\pi} \int d^2 \lambda \chi(\lambda) D(-\lambda).
\]  

(2.10)

In equation (2.10), \( D(\beta) = \exp (\beta a^\dagger - ^* a) \) is a Weyl displacement operator.

Now, the significance of the factorization (2.4) is quite transparent: it describes the superposition of the attenuated field with a thermal one whose time-dependent mean occupancy is \( \overline{n}_T(t) \) (Eq. (2.7)). Therefore, the decay of the field mode ruled by the quantum optical master equation is a thermalization process, as studied generically in our paper [12]. We can now use the generating-function method presented in reference [12] in order to obtain the time-dependent higher-order moments of the quadrature operators as functions of the similar ones at the initial time \( t = 0 \). Due to the oscillatory factors in the CF (2.4), such expectation values are also rapidly oscillating functions. In what follows, we give only formulae with the oscillatory factors removed. Application of the steps described in reference [12] to the factorization (2.4) yields the following formulae, valid for an even order \( N \) and \( j = 1, 2 \):

\[
\langle (\Delta X_j)^N \rangle_t = N! \sum_{m=0}^{[N/2]} \frac{[\overline{n}_T(t) + 1]^m \exp (-N^2 - 2m \gamma t)}{2^{2m} m!(N - 2m)!} \times \langle (\Delta X_j)^{N - 2m} \rangle_0,
\]  

(2.11a)

\[
\langle (\Delta X_j)^N \rangle_t = N! \sum_{m=0}^{[N/2]} \frac{[\overline{n}_T(t) + 1]^{-m} \exp (-N^2 - 2m \gamma t)}{2^{2m} m!(N - 2m)!} \times \langle (\Delta X_j)^{N - 2m} \rangle_0,
\]  

(2.11b)

\[
\langle (\Delta X_j)^N \rangle_t = N! \sum_{m=0}^{[N/2]} \frac{[\overline{n}_T(t)]^m \exp (-N^2 - 2m \gamma t)}{2^{2m} m!(N - 2m)!} \times \langle (\Delta X_j)^{N - 2m} \rangle_0,
\]  

(2.11c)

\[
\langle (\Delta X_j)^N \rangle_t = N! \sum_{m=0}^{[N/2]} \frac{[\overline{n}_T(t) - 1]^m \exp (-N^2 - 2m \gamma t)}{2^{2m} m!(N - 2m)!} \times \langle (\Delta X_j)^{N - 2m} \rangle_0.
\]  

(2.11d)

Here \( X_1 := (a + a^\dagger)/2 \) and \( X_2 := (a - a^\dagger)/2 \) are the quadrature operators, \( \Delta X_j := X_j - \langle X_j \rangle \), and the symbol \( \langle \rangle \) denotes the normal-ordering operation. We briefly examine the structure of these equations in connection with the concept of higher-order squeezing [19].

2.1 Intrinsic higher-order squeezing

As \( \langle (\Delta X_j)^N \rangle \) is always positive for even \( N \), from equation (2.11d) we learn that the condition for intrinsic