On the existence of an increasing symmetric equilibrium in \((k+1)\)-st price common value auctions

Ilia Tsetlin · Aleksandar Saša Pekeč

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Abstract In a classical result, Milgrom (1981a) established that the Monotone Likelihood Ratio Property (MLRP) is a sufficient condition for the existence of an increasing symmetric equilibrium in \((k+1)\)-st price common value auctions. We show: (1) If MLRP is violated, then for any number of bidders and objects there exists a distribution of the common value such that no increasing symmetric equilibrium exists; (2) If MLRP is violated, then for any distribution of the common value there exist infinitely many pairs of the number of bidders and the number of objects such that an increasing symmetric equilibrium does not exist; (3) There are examples where an increasing symmetric equilibrium exists even when the signal distribution violates MLRP.

Keywords Auctions · Symmetric equilibrium · Common value · Auction theory

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1 Introduction

Auction models provide an important tool for studying competitive markets. A critical property of an auction model is the existence of equilibrium. As Jackson and Swinkels (2005) note, “Much of what is known about existence of equilib-
Equilibrium in auctions comes from ... relying on monotonicity arguments (e.g., Athey (2001) or Maskin and Riley (2000)). Jackson (2005) shows that equilibria might fail to exist when bidders’ types are two-dimensional, i.e., when it is impossible to order bids of agents by their types.

Suppose that bidders’ types are one-dimensional. Would then an increasing symmetric equilibrium (where bids are increasing in bidders’ types) exist? We address this question in the standard common value auction model of Milgrom (1981a), where each bidder independently observes a signal about the common value. Milgrom (1981a) established that the Monotone Likelihood Ratio Property (MLRP) is a sufficient condition for the existence of an increasing symmetric equilibrium in \((k+1)\)-st price common value auctions. We consider whether MLRP is a necessary condition for the existence of an increasing symmetric equilibrium.

Our results are the following: MLRP is indeed a necessary condition, in the sense that for any signal distribution that violates MLRP one can either choose a distribution of the common value or choose the number of bidders and objects so that no increasing symmetric equilibrium exists. However, we also provide examples where an increasing symmetric equilibrium exists even though the signal distribution violates MLRP.

\section{The model}

Consider a standard model of \((k+1)\)-st price common value auctions. There are \(k\) identical objects for sale to \(n\) risk neutral bidders, with \(1 \leq k < n\). Each object has the same common value \(V\) to all bidders, drawn from the distribution with the probability density function (hereafter, pdf) \(g(v)\) and support \([v, \bar{v}]\).

Bidder \(i\), \(i = 1, \ldots, n\), independently observes signal \(X_i\) from a distribution with pdf \(f(x|v)\), cumulative distribution function (hereafter, cdf) \(F(x|v)\), and bids for one object. The objects are allocated to the \(k\) highest bidders and each of them pays the value of the \((k+1)\)-st (highest rejected) bid. We assume that \(g(v)\) is continuous and that fourth-order partial derivative of \(f(x|v), f_{xvvv}(x|v)\), exists. The latter is a technical assumption used in the proof of Lemma 3.1 to create bounds via third-order Taylor expansion.

To derive a symmetric equilibrium, it is useful to take the point of view of one of the bidders, say bidder 1 with signal \(X_1 = x\). Let \(Y_{n-1}^k\) denote the \(k\)-th order statistic from the set of the remaining \(n-1\) signals. Assume that \(b^* : \mathbb{R} \to \mathbb{R}\) is a (strictly) increasing symmetric equilibrium bidding function, so that \(b^{*-1}\) is well defined.

The following expression plays a key role in deriving an increasing symmetric equilibrium:

\[
v(x, y) = E\left[ V | X_1 = x, Y_{n-1}^k = y \right].
\]  \hspace{1cm} (1)

If bidder 1 bids \(b\), her expected payoff, conditional on observing signal \(X_1 = x\), is