Implementation of the Walrasian correspondence by market games

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Abstract. In this paper we present a set of axioms guaranteeing that, in exchange economies with or without indivisible goods, the set of Nash, Strong and active Walrasian Equilibria all coincide in the framework of market games.

1 Introduction

In this paper we present an unifying framework for the study of market games in exchange economies. In classical exchange economies when all goods are divisible particular market games have been considered by Shubik (1977) and Schmeidler (1980). A general axiomatic framework was introduced by Benassy (1986). He proposed a set of axioms that defines a market game, and he was able to show that 1) the Walrasian allocation can be supported as a Nash Equilibrium (NE in the sequel), and 2) that when all markets are active, all NE yield Walrasian allocations. It was shown in specific models by Dubey (1982) (assuming divisible goods) and Svensson (1991) (assuming one indivisible good) that the set of Strong Equilibrium (SE) outcomes coincides with the set of NE outcomes when all the markets are active and with the set of Walrasian allocations (other models of indivisibilities include those of Roth 1982; Kaneko 1983; Gale 1984; Quinzii 1984; Svensson 1984, 1988; Demange and Gale 1985; Maskin 1987; Roth and Sotomayor 1988; Tadenuma and Thomson 1990).

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A natural question is whether the results explained above depend on either the special structure of the commodity space or on the particular game form used. In this paper we address this question and show that neither is the case. We present a model in which the commodity space has no special structure (and thus it could have both many divisible and many indivisible goods), and we propose a set of axioms that a market game should satisfy. With these axioms in hand we prove that the set of Walrasian allocations, SE outcomes and NE outcomes when all markets are active coincide.

The main insight of this paper is that from the point of view of implementation in NE and SE, convexity does not matter very much. Other insights gained by the study of market games are the following: 1) To sustain Walrasian allocations as NE or SE equilibria outcomes requires relatively harmless and simple assumptions. 2) However, to eliminate non-Walrasian equilibria requires strong assumptions on the mechanism (i.e., some kind of Bertrand competition which implies that the outcome function is discontinuous, see Benassy 1986, pp. 99–100) and the assumption that all markets are active. 3) Individual feasibility cannot be guaranteed outside equilibrium.1 And 4) as long as all markets are active, the fact that coalitions can or cannot be formed does not matter since implementation occurs in NE and SE. Given that Implementation by means of market games has intuitive appeal because the underlying mechanism is both natural and simple, it would be nice to know if our results can be extended to production economies and other social choice rules.

Our work can be regarded as an application of implementation theory to a specific problem. The authors hope that it might modestly contribute to show that implementation is not only about abstruse mechanisms but can also shed some light on the functioning of real life institutions like double auction markets. It should be remarked though that market games mimic but are not markets (in the same way than an artificial heart mimics but is not a heart). In particular some tendencies that are spontaneous in a market are reflected here in the form of the outcome function. For instance, in a frictionless market arbitrage implies that there is only one trading price for each good. In the class of market games considered here, the existence of an unique trading price is assumed directly.

The rest of the paper goes as follows. In the next section we present our model of a market game. Section 3 gathers our main results. Finally, Sect. 4 offers some comments.

2 The model

This section has five parts. a) The description of the environment, b) the definition of a social choice correspondence, c) the description of a game form (also called a mechanism), d) the definition of the game-theoretical equilibrium concepts and e) the notion of implementation.

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1 For the issue of Individual Feasibility see our comments after Theorem 1. See also Postlewaite and Wettstein (1989).