Closing the case on closure in Cole’s model

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Abstract. In his response to Closure in Cole’s Reformulated Leontief Model (Jackson et al. 1997) Cole presents the mechanics of his solution. His spreadsheet demonstration of the model, however, fails to fully address the conceptual issues and model specification concerns raised in our critique. His demonstration does enable us to provide a sorely needed formalization that pinpoints the inconsistencies in his model. While we join with Cole in urging modellers to strive for clarity, transparency, and utility, we caution against allowing these goals to cause us to lose sight of internal consistency requirements.

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1 Introduction

In his response to our critique, Cole (1997) uses a spreadsheet version of his lagged expenditure model to demonstrate how his empirical results are obtained. He succeeds in describing a set of steps that generate what might appear to be reasonable numerical values, but his response falls short of providing a transparent mathematical expression of the model. A rigorous and comprehensive assessment of the internal consistency and logic underlying the lagged expenditure model is not possible in the absence of such a formal statement. The following section explicates the elusive mathematical foundation for Cole’s method, derived directly from the spreadsheet version presented in his response. Although the spreadsheet example generates numerical values, the formal solution identifies the remaining inherent inconsistencies. The final section provides a summary and concluding remarks.
2 Cole’s approach formalised

We agree with Cole that when the temporal distribution of impacts is considered in a properly specified model, the matrix of coefficients will approach the traditional coefficient matrix from below. Thus, for any time horizon shorter than that necessary to overcome all production lags, the corresponding coefficients will be some fraction of their long run values. Since this was never in question, we set aside the issue of temporal lags to focus directly on the system of equations that forms the basis of Cole’s model.

The conventional Leontief inverse is the mechanism by which output responses to changes in final demand are determined. Because the inverse is the solution to a set of simultaneous equations, we begin by identifying the system of equations for which Cole’s inverse matrix is the solution. In Cole’s system,

\[
\begin{align*}
x_{ss} + x_{sy} + dx_{sr} + (1 - d)x_{sr} &= X_s \quad (1) \\
x_{ys} + x_{yy} + dx_{yr} + (1 - d)x_{yr} &= X_y \quad (2) \\
x_{rs} + x_{ry} + dx_{rr} + (1 - d)x_{rr} &= X_r \quad (3)
\end{align*}
\]

where \(s\) denotes what Cole describes as supply, \(y\) denotes demand, \(r\) denotes “Rest of World” (RoW) and \(d\) is Cole’s feedback parameter, representing the proportion of RoW demand that feeds back to the regional economy per annum.

The fourth left-hand-side terms from the three equations necessarily constitute final demand, since they form the “exogenous” part of the system. We note that the exogeneity is determined by temporal rather than spatial or economic characteristics, as is more usual in the standard input-output model. Hence, and of utmost importance, this final demand is not independent of \(x_{ij}\), as is the case in the normal transaction matrix. Given this relationship, we can convert to coefficient form and, letting \((1 - d)x_{ir} = f_i\), write:

\[
\begin{align*}
a_{ss}X_s + a_{sy}X_y + da_{sr}X_r + f_s &= X_s \quad (4) \\
a_{ys}X_s + a_{yy}X_y + da_{yr}X_r + f_y &= X_y \quad (5) \\
a_{rs}X_s + a_{ry}X_y + da_{rr}X_r + f_r &= X_r \quad (6)
\end{align*}
\]

and therefore \(A^dX + F = X\) where:

\[
A^d = \begin{bmatrix}
a_{ss} & a_{sy} & da_{sr} \\
a_{ys} & a_{yy} & da_{yr} \\
a_{rs} & a_{ry} & da_{rr}
\end{bmatrix}, \quad (7)
\]

Although we use terms like ‘conventional’ and ‘normal’ models in this document to distinguish Cole’s framework from standard input-output, we are in no way suggesting that analysts should adhere only to standard forms of modelling frameworks. To suggest, as Cole does, that we take an overly restrictive view of input-output analysis (Cole 1997, p.34 and p.39) is curious, given our collective contributions to the literature on extended and alternative input-output accounting frameworks (Jackson 1986a–c, 1988, 1989; Dewhurst and Madden 1998; Madden 1985,1988, 1993; Madden et al. 1996; Madden and Trigg 1990; Trigg and Madden 1990, 1995).