A Modified Binomial Tree Method
for Currency Lookback Options

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Abstract The binomial tree method is the most popular numerical approach to pricing options. However, for currency lookback options, this method is not consistent with the corresponding continuous models, which leads to slow speed of convergence. On the basis of the PDE approach, we develop a consistent numerical scheme called the modified binomial tree method. It possesses one order of accuracy and its efficiency is demonstrated by numerical experiments. The convergence proofs are also produced in terms of numerical analysis and the notion of viscosity solution.

Keywords Modified binomial tree method, Currency lookback options, Convergence

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1 Introduction

Lookback options are path dependent options whose payoffs depend on the maximum or the minimum of the underlying asset price during the life of the options (see [1–3]). Valuation formulas have been produced for European lookback options by Goldman, Sosin and Gatto (1979) [4]. Just like vanilla options, generally speaking, analytical valuation formulas are not available for American lookback options. As a result, numerical methods should be adopted to price American lookback options.

The binomial tree method, as a discrete model proposed by Cox, Ross and Rubinstein (1979) [5], is the most popular numerical approach to pricing options. Hull and White (1993) and Barraquand and Pudet (1996) proposed a binomial tree method for lookback options, separately [6,7]. Cheuck and Vorst (1997) suggested an equivalent but simple algorithm [8]. Dai (1999) established the equivalence of the binomial tree method and an explicit difference
scheme, and proved the convergence of the binomial tree method for American lookback call options based on the notion of viscosity solution [9].

In this paper we focus on currency lookback options, since lookback options are mostly structured with a foreign exchange rate as an underlying variable. In the over-the-counter market, such options are traded with continuously monitoring. It has been observed by many researchers that the binomial tree method for currency lookback options converges very slowly. Cheuck and Vorst (1997) pointed out that this method is for discrete monitoring [8]. Later we will show that, in the viewpoint of PDE, the binomial tree method is not consistent with the continuous models for continuously monitored lookback options. This leads to the slow speed of convergence of the binomial tree method.

The purpose of this paper is to develop a more efficient scheme. On the basis of the PDE approach, we propose a consistent scheme that we call the modified binomial tree method since it can be regarded as a slight modification of the binomial tree method. This method possesses one order of accuracy and its efficiency is demonstrated by numerical experiments. Numerical results show that the modified binomial tree method presented here prevails over the binomial tree method in all cases.

In addition, we show that the values of currency lookback options computed from the modified binomial tree method converge to the true solutions of the corresponding continuous model. In fact, due to consistency, the convergence can be directly deduced by the Lax theorem for European cases. For American currency lookback call options, the proof presented in [9] also applies to the modified binomial tree method. But, for American currency lookback put options, it would cause some difficulties. The key point is to obtain uniform estimates of the bounds of the approximate solutions sequences computed from the modified binomial tree method.

This paper is organized as follows: in the next section we recall the continuous models for lookback options; in Section 3 we recall the binomial tree method and point out that the binomial tree method for lookback options is not consistent with the continuous models; in Section 4 we present the modified binomial tree method; Section 5 is devoted to the convergence proofs; lastly numerical experiments are given.

2 Continuous Models

A (floating strike) lookback call gives the holder the right to buy at the lowest realized price while a (floating strike) put allows the holder to sell at the highest realized price over the lookback period. Throughout the rest of this paper, we consider only the currency lookback put option. The case of currency lookback call is similar.

Let $T$ be the time of expiration and $[0, T]$ the lookback period. Suppose the foreign exchange rate process $(S_t)_{0 \leq t \leq T}$ is governed by the following stochastic differential equation: $\frac{dS_t}{S_t} = \mu dt + \sigma dB_t$, where $(B_t)_{0 \leq t \leq T}$ is a standard Brownian motion, defined on some probability space, $\mu$ is the expected return rate and $\sigma$ is volatility. Denote the maximum foreign exchange rate from 0 to $t$ ($t \in [0, T]$) by $M = \max_{0 \leq \tau \leq t} S_\tau$. Let $P = P(S, M, t)$ stand for option price.