Technical Article

A Finite Element Model to: 1. Predict Groundwater Inflow to Surface Mining Excavations

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Abstract. Prediction of groundwater inflow into an excavation is important during the feasibility stage of a surface mine. A numerical finite element model called SEEP/W was used to predict the inflow of water and to calculate the height of the seepage face in and around the pit. The results are compared to those from analytical solutions as well as other numerical codes for estimating groundwater inflow into a surface mine. Mine operators can use the results of such models to design dewatering systems.

Key words: Backfilled open cut mine, finite element method, Galerkin approach, groundwater inflow

Introduction

Surface mining operations below the groundwater table can cause excessive inflow to the mine and serious water-related problems. The inflow from the surrounding strata towards the pit requires installation of dewatering facilities to keep the workings dry and create an extensive and prolonged cone of depression. In order to design an effective drainage scheme for a surface mine, prediction of water inflow is an important component of the feasibility stage of the operation. Analytical and numerical models have been used to estimate water inflow into surface mines (McWhorter 1981; Singh and Atkins 1984, 1985a, 1985b; Singh et al. 1985; Singh and Reed 1987; Williams et al. 1986; Domenico and Schwartz 1990; Rubio and Lorca 1993; Hanna et al. 1994; Davis and Zabolotney 1996; Azrag et al. 1998; Lewis 1999; Marinelli and Niccoli 2000).

Analytical solutions are based on assumptions and special boundary conditions that limit their applicability and are not as versatile as numerical methods, which can better deal with complex mining situations. For example, most analytical solutions do not directly account for inflow through the bottom of the pit (Hanna et al. 1994). Although the analytical solutions developed by Marinelli and Niccoli (2000) account for upward flow through the bottom of the excavation, the solutions assume an equivalent porous medium and are based on many simplifying assumptions. Hence, analytical solutions are not appropriate for all hydrogeological situations.

Numerical models can simulate all aquifer conditions and can provide a more realistic representation of the interaction between groundwater systems and mining excavations. However, many numerical models are limited in their ability to represent the variable height of the seepage faces and cannot easily model various aquifer conditions such as saturated/unsaturated flow, confined/unconfined flow and non-linear hydraulic characteristics of the porous medium. In order to overcome some of these limitations, the authors have used a two-dimensional saturated/unsaturated, unconfined, transient finite element model called SEEP/W (Geo-slope International Ltd 2002) to predict inflow into surface mining excavations.

Capabilities of the SEEP/W Model

The SEEP/W model can simulate both saturated and unsaturated flow. The ability to assume unsaturated flow conditions allows the model to solve a wider range of problems and to obtain realistic inflow results. The model is able to simulate seepage faces on highwalls, predicting both the configuration of the phreatic surface and the height of the seepage face on the highwalls, thus providing the information needed for slope stability analysis. One of the great features and capabilities of the model is definition of the hydraulic conductivity and volumetric water content as a function of pore-water pressure in saturated-unsaturated flow systems. The model simulates heterogeneous hydraulic properties such as hydraulic conductivity and storage in an isotropic and heterogeneous flow system. A conductivity function, which defines the relationship between hydraulic conductivity and pore-water pressure can be defined for each material. This feature is very important in simulating groundwater inflow in backfilled open cut mines where hydraulic characteristics of the spoil are different from those of the unmined aquifer and unexcavated rocks. Furthermore, the horizontal hydraulic conductivity differs considerably from the vertical hydraulic conductivity in heterogeneous systems. The model can take transient boundary conditions into consideration and the user is able to modify boundary conditions in response to predicted results. It can also simulate unconfined flow by assuming zero pore-water pressure contours within
the flow domain in order to represent the water table. Although many numerical models are now used to predict groundwater flow into a surface mine, they are limited in their ability to quantify more detailed problems by taking the above-mentioned features and relationships into account. The SEEP/W finite element model was used to overcome most existing modelling problems.

**Governing Equation of the Inflow Model**

The governing partial differential equation for two-dimensional saturated/unsaturated flow of ground water can be obtained by coupling the continuity equation and Darcy’s law (Freeze and Cherry 1979):

\[ \frac{\partial}{\partial x} \left( K_x \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial H}{\partial y} \right) - \frac{\partial}{\partial t} (\theta) + Q = 0 \]  

(1)

where \( K_x \) and \( K_y \) are the hydraulic conductivities in the \( x \) and \( y \) directions respectively, \( Q \) is the recharge or discharge per unit volume, \( H \) is the hydraulic head, \( t \) is the time and \( \theta \) is the volumetric water content or moisture content. A change in moisture content (\( \theta \)) may be related to a change in pore-water pressure followed by a change in the total hydraulic head using the following equation (Freeze and Cherry 1979):

\[ \frac{\partial \theta}{\partial t} = C_{uw} \frac{\partial H}{\partial t} \]  

(2)

where \( C_{uw} \) is the slope of the water storage curve.

Substituting Equation 2 into Equation 1, the following general differential equation is obtained:

\[ \frac{\partial}{\partial x} \left( K_x \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial H}{\partial y} \right) - C_{uw} \frac{\partial}{\partial t} (H) + Q = 0 \]  

(3)

Equation 3 was used to predict groundwater inflow to an open cut mine.

**Solution Method**

To solve Equation 3 by the finite element method, the Galerkin approach was used to determine an approximate solution. (Further details on the Galerkin formulation can be found in Pinder and Frind 1972; Pinder et al. 1973; Pinder 1973; Neuman 1974; Pickens and Lennox 1976; Gray and Pinder 1974; Rabbani 1994). Using the Galerkin approach, the general finite element equation for groundwater flow can be written in a matrix form:

\[ [A]\{h\} + [B] \left\{ \frac{dh}{dt} \right\} + [C] = 0 \]  

(4)

where \([A]\) and \([B]\) are \( n \) by \( n \) matrices with \( n \) being the number of nodes in which:

\[ A_{ij} = \int_{\Omega} \left[ K_x \frac{dN_i}{dx} \frac{dN_j}{dx} + K_y \frac{dN_i}{dy} \frac{dN_j}{dy} \right] \, dx \, dy \]  

(5)

\[ B_{ij} = \int_{\Omega} m_w \gamma_w N_i N_j \, dx \, dy \]  

(6)

\([C]\) is the flux vector in which

\[ C_i = -\int_{\partial \Omega} Q N_i \, dS - \sum_{j=1}^{n} \left[ K_x \frac{dN_j}{dx} t_i + K_y \frac{dN_j}{dy} t_j \right] h_i \, d\beta \]  

(7)

where \( N_i \) are the basis functions or the interpolating functions. The last term in Equation 7 incorporates the Neumann boundary condition in the form:

\[ \int_{\partial \Omega} N_i \, q \, d\beta \]  

(8)

where \( q \) is the flux of water and can be expressed as:

\[ q = K \frac{dh}{dt} \]  

(9)

This term is formed only when groundwater flux is not zero along the edge of a boundary element.

**Finite Element Discretization Method**

The suitability of the Galerkin approach depends on the choice of basic functions. We divided the cross-section of the flow domain into quadrilateral elements, using a suitable number and arrangement of elements so that a realistic solution could be adequately approximated. The integrals that appear in the matrices of Equation 4 must be evaluated using numerical methods. A Gaussian quadrature scheme can be used to perform integrations (Pinder and Frind 1972; Wang and Anderson 1982) and an exact solution can be obtained (Pinder 1973). Integrations are carried out in the local coordinate system \((\xi, \eta)\), with limits of integration between -1 and +1, and then transformed to the corresponding global coordinates system \((x, y)\).

**Time Integration Technique**

Equation 4 is a first-order matrix differential equation. To solve for the undetermined coefficients, \( H_i (i = 1, 2, ..., n) \), a finite difference approximation scheme can be introduced into Equation 6. Pinder (1973) found that a backward difference scheme provided a more accurate groundwater flow solution than a Crank-Nicholson centered scheme in time approach. Using this approach (Pinder 1973; Geo-slope International Ltd 2002) yields: