Myocardial material parameter estimation
A non–homogeneous finite element study from simple shear tests

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Abstract  The passive material properties of myocardium play a major role in diastolic performance of the heart. In particular, the shear behaviour is thought to play an important mechanical role due to the laminar architecture of myocardium. We have previously compared a number of myocardial constitutive relations with the aim to extract their suitability for inverse material parameter estimation. The previous study assumed a homogeneous deformation. In the present study we relaxed the homogeneous assumption by implementing these laws into a finite element environment in order to obtain more realistic measures for the suitability of these laws in both their ability to fit a given set of experimental data, as well as their stability in the finite element environment. In particular, we examined five constitutive laws and compare them on the basis of (i) “goodness of fit”: how well they fit a set of six shear deformation tests, (ii) “determinability”: how well determined the objective function is at the optimal parameter fit, and (iii) “variability”: how well determined the material parameters are over the range of experiments. Furthermore, we compared the FE results with those from the previous study.

It was found that the same material law as in the previous study, the orthotropic Fung-type “Costa-Law”, was the most suitable for inverse material parameter estimation for myocardium in simple shear.

1 Introduction
Cardiovascular disease is a frequent cause of death and morbidity world-wide (Reddy and Yusuf 1998). The mechanical properties of ventricular muscle substantially influence the pumping function of the heart. The mechanical properties of passive myocardium play a major role during the diastolic filling phases of the heart cycle. Stiffening of the myocardium leads to impaired filling, which can lead to increased filling pressure, increased cardiac work, and ultimately decreased pump function via the Frank-Starling mechanism. Such diastolic dysfunction is often associated with heart failure and may be observable before appreciable evidence of systolic dysfunction (Mandilov et al. 2000). Thus, understanding of the passive mechanical properties of the myocardium is central to the understanding of these disease processes.

Early mathematical models of the whole heart focussed on the relationship between blood pressure and cavity volume, which has been used by clinicians for many years (Suga et al. 1973; Janicki and Weber 1977). In recent decades, it has become apparent that an improved understanding of regional variation of myocardial tissue properties is important to understand the fundamental mechanisms underlying ventricular mechanics, such as wall thickening and shearing deformations. The apparent heterogeneous, anisotropic mechanical properties have since been represented using a variety of material laws (ML) based on a different theoretical frameworks, ranging from elastic to viscoelastic, and from phenomenological to microstructurally based approaches. It has been demonstrated that myocardial tissue exhibits a non-elastic response (Dokos et al. 2002). Several studies have fitted rheological constitutive relations to viscoelastic data (Bischoff et al. 2004; Miller and Wong 2000), however, at present there are insufficient data on the viscoelastic properties of passive ventricular myocardium at the physiological
strain rates occurring in vivo, particularly in shear modes of deformation (relative to the laminar architecture). Many studies have focussed the analysis to the hyperelastic properties, with particular attention paid to anisotropy.

Guccione et al. (1991) modelled the equatorial region of the canine left ventricle as a thick–walled cylinder consisting of an incompressible transversely isotropic exponential Fung-type hyperelastic material (Fung 1965, 1993). Subsequently, LeGrice et al. (1995a) showed that the microstructure is a composite of discrete layers of myocardial fibres of usually four to six cells thick, which suggested an orthotropic mechanical response. To this end, the transversely isotropic Fung-type relation was extended to account for orthotropy by Fung-type hyperelastic material (Fung 1965, 1993). Subsequently, (2000).

Various studies have shown the significance of shear deformation in cardiac mechanics, (Arts et al. 2001; LeGrice et al. 1995b) and the shear properties of passive ventricular myocardium remain poorly characterized. We recently compared a number of myocardial constitutive relations with the aim to extract their suitability for inverse material parameter estimation (Schmid et al. 2006). In the context of three dimensional simple shear experiments from Dokos et al. (2002), and assumed a homogeneous deformation. In this study, we relax this assumption, by using finite element analysis methods to account for the non-homogeneous deformations that occur during simple shear due to the Poynting-effect (Poynting 1909). This allowed us to compare the finite element results with our previous homogeneous results (Schmid et al. 2006). This comparison is very useful for experimentalists who are generally concerned with the assumption of homogeneity in simple shear and biaxial tests, e.g. Gardiner and Weiss (2001).

In this study, we compared five constitutive laws:

1. Costa law (CL)
2. Separate Fung-type law (SFL)
3. Pole–zero law (PZL)
4. Tangent law (TL)
5. Langevin Eight-chain law (LECL).

Further details of these material laws are presented in the next section. SFL and TL were designed to have similar features to CL and PZL, respectively, and were investigated to determine whether any improvement could be obtained over the latter relations. These constitutive relations provide characterizations of experimental data and material properties that are useful in practice. Note that theoretically desirable properties such as polyconvexity and fibre dispersion, etc. (Itskov and Aksel 2004; Gasser et al. 2006; Leonov 2000; Lainé et al. 1999) were not accounted for in the design of these material laws and will be the subject of future studies.

To compare parameter estimation results for the different laws, we examined three major criteria. “Goodness of fit” is the ability of a material law to minimize a given objective function. This is quantified by two measures being the relative error of the estimate, and the Akaike Information Criterion (AIC) (Burnham and Anderson 2002), which penalises the number of material parameters in each model. “Determinability” quantifies how well the material parameters are determined at the optimum by means of optimality criteria, see Lanir et al. (1996). Parameter “Variability” over the range of experiments for a given material law was also examined.

We start this report by presenting the theoretical background for the material parameter estimation process (i.e. the details of five material laws, and their similarities), a finite element implementation, a model convergence analysis, and the objective function we used. This is followed by a descriptive summary of the three comparison measures, the results of the numerical studies, and the subsequent statistical analysis. We also compare these finite element study results with the results from our previous homogeneous simulations (Schmid et al. 2006). Finally, we end with a discussion of the advantages and limitations of the various constitutive relations.

2 Methods

We first briefly summarize the fundamentals of continuum mechanics. The deformation is described by the deformation gradient tensor \( \mathbf{F} \). The strain is quantified using the Green strain tensor \( \mathbf{E} = \frac{1}{2} (\mathbf{F}^T \mathbf{F} - \mathbf{I}) \). The balance of linear momentum is expressed using Eq. (1), where \( S \) is the second Piola–Kirchhoff stress tensor, (Holzapfel 2000).

\[
\text{Div}(\mathbf{S}) = 0
\]

Assuming hyperelasticity the remaining relationship between the stress \( \mathbf{S} \) and the strain \( \mathbf{E} \) is then specified by a function, the strain energy density \( \Psi = \Psi(\mathbf{E}) \).

2.1 Tissue experiments

We base our modelling investigations on experimental data taken from Dokos et al. (2002). Passive shear properties of six pig hearts were examined. Samples (\( \sim 3 \times 3 \times 3 \text{ mm} \)) were cut from adjacent regions of the lateral left ventricular midwall, with sides aligned with the microstructural material axes \((f, n, s; \text{f}iber, \text{normal, sheet})\). Sinusoidal cycles of simple shear (shear displacement range \([-50\%, 50\%]\)) were applied separately to each specimen in two orthogonal directions. Three specimens from each heart were tested in two directions, giving all six possible modes of shear with respect to the microstructural axes. Data for the fitting of material