Investigation of the influence of reflection on the attenuation of cancellous bone

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Abstract The model proposed in this paper is based on the fact that the reflection might have a significant contribution to the attenuation of the acoustic waves propagating through the cancellous bone. The numerical implementation of the mentioned effect is realized by the development of a new representative volume element that includes an infinitesimally thin ‘transient’ layer on the contact surface of the bone and the marrow. This layer serves to model the amplitude transformation of the incident wave by the transition through media with different acoustic impedances and to take into account the energy loss due to the reflection. The proposed representative volume element together with the multiscale finite element is used to simulate the wave propagation and to evaluate the attenuation coefficient for samples with different effective densities in the dependence of the applied excitation frequency. The obtained numerical values show a very good agreement with the experimental results. Moreover, the model enables the determination of the upper and the lower bound for the attenuation coefficient.

Keywords Reflection · Attenuation · Multiscale FEM · Cancellous bone · Homogenization

1 Introduction

The cancellous bone is a tissue with a complex microstructure, consisting of the solid bone framework filled with the fluid marrow. It can be found inside the vertebrae but also as the internal part of the end regions of some other bones. Due to the specific composition, the behavior of the cancellous bone shows many interesting effects, which attract attention of researchers from different scientific fields. An important issue is, for example, the determination of material parameters of single trabeculae where the standard investigation techniques fail because of the extremely small size of the sample which should be examined. Dependent on the applied investigations technique, a wide spectrum of the possible values for these parameters can be found in the literature (Black and Hastings 1998; Cowin 1999; Laugier and Haïat 2011). Another interesting effect certainly is the separation of an incident wave into a fast and a slow wave. An analytical solution explaining this phenomenon was first presented in the famous work of Biot (1956a,b), which served as a basis for many alternative analytical approaches (Buchanan et al. 2004; Buchanan and Gilbert 2006; Fang et al. 2007; Cherkaev 2001; Bonifasi-Lista et al. 2009; Fellah et al. 2004; Sebaa et al. 2006). On the other hand, the experimental investigation of the wave separation started with the work of Hosokawa and Otani (1996). Since then, more and more precise measurements have become available due to the rapid development of technology (Chaffai et al. 2000a; Kabel et al. 1999; Pakula et al. 2009; Hosokawa and Otani 1998). The extremely high-attenuation coefficient of the cancellous bone is another topic worth mentioning. Apart from the bone mineral density (BMD), the attenuation is believed to be a further reliable indicator for the determination of the bone micro-architecture (Bossy et al. 2005; Haïat et al. 2008b; Ilic et al. 2010, 2011).
In the context of numerical modeling of cancellous bone, there are two approaches predominant. The finite differences time domain (FDTD) method has been used in many contributions for the simulation of 2D and 3D problems (Bossy et al. 2004, 2005; Hosokawa 2006; Haïat et al. 2007). This method belongs to the group of direct methods for the solution of differential equations. Due to its straightforward implementation, it is very suitable for extension of basic models by introducing different additional effects, such as scattering (Luppé et al. 2002; Bas et al. 2004; Padilla et al. 2008; Haïat et al. 2008a,b). Another common numerical approach is the finite element method (FEM) and especially its variant, the digital-image based FEM. In this method, the digital images of high resolution are transformed into the voxel-based FE models which are used for a further analysis (Arbenz et al. 2008; Rubin et al. 2010).

Within the scope of this contribution, the multiscale FEM has been chosen for the modeling of cancellous bone. This is a numerical approach, adapted to the modeling of heterogeneous material with a highly oscillatory microstructure (Schröder 2000; Miehe et al. 2002; Torquato 2002; Zohdi and Wriggers 2005; Ilic and Hackl 2009; Klinge and Hackl 2012; Oskay and Fish 2007; Fish and Kuznetsov 2010).

The multiscale FEM has already been used in the previous works (Ilic et al. 2010, 2011) and has shown the following important advantages: Due to the linear material behavior of the bone, the simulations do not require a high calculation time. Even if the representative volume element (RVE) varies in a macroscopic sample, the calculation can be accomplished within a realistic period of time. Another advantage of this concept certainly is the high flexibility in the choice of the problems, which can be simulated at different levels of the multiscale approach.

The two contributions mentioned (Ilic et al. 2010, 2011) were focused on the investigation of an optimal form for the RVE. For this purpose, a cube-formed sample consisting of the solid framework and the fluid marrow was considered. In the work by Ilic et al. (2010), it was assumed that the solid frame is composed of thin walls, consisting of shell elements, while the results presented in Ilic et al. (2011) are based on the assumption that the solid phase is made of thin columns, modeled by using solid elements. Both types of the RVE yield good results for mechanical properties, such as the Young modulus, the bulk modulus and the Poisson ratio. However, the values of the attenuation coefficient departed from the measured ones, and the modification of the solid framework structure—thin walls versus thin columns—did not significantly contribute to the change of the attenuation parameter.

The goal of the research work considered in this paper is to improve the previously proposed RVEs by taking the effects of reflection into consideration. This idea is substantiated by the well-known fact that the reflection—a phenomenon occurring due to the transition of the wave through media with different acoustic impedances—significantly contributes to the weakening of the propagating wave.

The paper is structured as follows. Section 2 provides an insight into a common way of cancellous bone investigation as well as into the main properties of the component materials, namely solid bone and fluid marrow. Section 3 summarizes basic information on the multiscale concept, including details on the derivation of the boundary conditions at the microlevel. The elements used for the FE model are presented in Sect. 4. The choice encountered in this contribution is concerned with the formulation of the solid elements extended for the purpose of simulations in the complex domain. The remaining part of the paper is focused on the extension of the model by the implementation of reflection effects. The physical background of this phenomenon is explained in Sect. 5, while its variational formulation, necessary for the derivation of a new type of element within the FEM approach, is derived in Sect. 6. The variational formulation is based on the definition of the power of a wave as well as on the jump-formed change of the amplitudes of the incident and transferred wave. The numerical implementation of the mentioned effects, explained in Sect. 7, is based on the idea of the ‘transient’ layer of the infinitely small thickness where the transformation of the wave amplitudes occurs. The same concept is also convenient for the implementation of the friction effects due to the relative motion of oscillating particles at the contact of different phases, which is documented in the Appendix. The numerical results, presented in Sect. 8, are considered with the simulation of the wave propagation through the samples of different effective densities. The influence of the frequency of the excitation is also taken into account in the simulations. A comparison of the results obtained with and without considering reflection effects is used to quantify the influences of the newly introduced phenomenon. In the end, the paper draws several conclusions which also include a discussion concerning the possibilities of a further optimization of the prescribed model.

2 Properties and investigation of cancellous bone

In order to explain and motivate the assumptions introduced in the model, a typical laboratory test will be explained at first (Williams and Johnson 1989; Hosokawa and Otani 1996). During such a test, a bone sample of the approximate dimensions 2.5 × 2.5 cm and with a thickness of 7–10 mm is placed in a water bath between two piezoelectric transducers (Fig. 1). The first transducer emits sound waves and the second one records the signals after their propagation through the water and bone sample. The measurements are repeated for samples with different thicknesses, and the recorded signals are used to estimate the effective material properties.