Analysis of the $M^{[X]}/G/1$ Queues with Second Multi-optional Service and Unreliable Server

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Abstract A bulk-arrival single server queueing system with second multi-optional service and unreliable server is studied in this paper. Customers arrive in batches according to a homogeneous Poisson process, all customers demand the first “essential” service, whereas only some of them demand the second “multi-optional” service. The first service time and the second service all have general distribution and they are independent. We assume that the server has a service-phase dependent, exponentially distributed life time as well as a service-phase dependent, generally distributed repair time. Using a supplementary variable method, we obtain the transient and the steady-state solutions for both queueing and reliability measures of interest.

Keywords Bulk-arrival queue, first essential service, second optional service, reliability

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1 Introduction

In a recent paper, Madan$^2$ considers an $M/G/1$ queue with the second optional service, in which some of the customers may require immediately after completion of the first essential service. Such queueing situations can be found in day to day life. By using a supplementary variable method, Madan$^2$ studies the time-dependent as well as the steady state behavior of this kind of queueing system. Based on Madan’s model, some authors propose various modifications. Medhi$^3$ considers a more general case of such a queue, where the service times of all service phases are independent and have general distributions. Yin et al.$^6$ generalize the model with assumptions that customers arrive at the system in batches according to a Poisson process, and the customer may opt for a second multi-optional services when the first essential service is completed. Wang$^5$ studies Madan’s model from the viewpoint of reliability theory with the assumption that the server may be subject to breakdowns and repairs during the service processes.

The present study focuses on generalizing the above works in an integrated way, i.e., the $M^{[X]}/G/1$ queueing system with server breakdowns and repairs is studied in this paper. Such queueing situations are also common in practice. For instance, in flexible manufacturing system, there are versatile, multi-functional machines which can perform several types of operations, e.g., lathing, drilling, milling and so forth. Workpieces arrive in batches with different processing requirements, all need the main essential service, whereas some of them may require further particular type of operation after the main essential service. Meanwhile, the machine may be...
subject to random failures and/or interruptions which have impact on the system’s performance. We wish to understand their effects on measures of system performance which influence the efficient operation of the systems.

The rest of this paper is organized as follows. In the next section, we give a relative formal description of the model and introduce supplementary variables to make the process Markovian. In Section 3, we investigate the time-dependent solutions to this model by using the Laplace transforms technique. In Section 4, the steady state solutions for the queueing quantities are obtained and it is shown that some previous works are special cases of our model. In Section 5, we consider the reliability aspect of the model and obtain some important reliability measures of the server, including the server availability, failure frequency, and the reliability function. Finally, in Section 6, we work out some numerical examples to illustrate the effect of the unreliability parameters on the system performance.

2 Basic Assumptions and Description of the Model

We consider the \( M[X]/G/1 \) queueing systems with the following structure.

1. Customers arrive at the system according to a Poisson process with rate \( \lambda > 0 \) and arrive in batches such that the batch size \( X \) are i.i.d. random variables with distribution \( P(X = i) = C_i, \ i = 1, 2, \cdots, \) with mean \( c = EX \) and \( EX^2 < \infty. \)

2. The first essential service is needed by all arriving customers. Let \( B_0(v) \) and \( b_0(v) \), respectively, be the distribution function and the density function of the first service times \( V_0 \), with mean \( 1/\mu_0 \), and let \( \mu_0(x) = b_0(x)/B_0(x) \) be the hazard rate function. It is assumed that \( EV_0^2 < \infty. \)

3. As soon as the first service of a customer is completed, then with probability \( r_k(1 \leq k \leq m) \) he may opt for the \( k \)-type second service, in which case his second service will immediately commence or else with probability \( r_0 = 1 - \sum_{k=1}^{m} r_k \) he may opt to leave the system, in which case another customer at the head of queue (if any) is taken up for his first essential service.

4. The \( k \)-type second service times \( V_k \) are assumed to be generally distributed with distribution function \( B_k(v) \), density function \( b_k(v) \), hazard rate function \( \mu_k(x) = b_k(x)/1-B_k(x) \) with mean service time \( 1/\mu_k \), \( 1 \leq k \leq m \). It is assumed that \( EV_k^2 < \infty. \)

5. We assume that the server’s life time has exponential distribution with mean \( 1/\alpha_0 \) in the first essential service. In the \( k \)-type second optional service, the server fails at an exponential rate \( \alpha_k \) \( (1 \leq k \leq m) \).

6. The server may break down when servicing customers, and when the server breaks down it is sent for repair immediately. The customer just being served before server breakdown waits for the server to complete its remaining service. The repair times, \( W_k(0 \leq k \leq m) \), of both service phases are arbitrarily distributed with probability distribution function \( G_k(x)(0 \leq k \leq m) \). Also, let \( g_k(x), \beta_k(x) \) and \( 1/\beta_k(0 \leq k \leq m) \), be the corresponding probability density functions, hazard rates functions, and means, respectively.