Fast reduction of undersampling artifacts in radial MR angiography with 3D total variation on graphics hardware

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Abstract  Objective  Subsampling of radially encoded MRI acquisitions in combination with sparsity promoting methods opened a door to significantly increased imaging speed, which is crucial for many important clinical applications. In particular, it has been shown recently that total variation (TV) regularization efficiently reduces undersampling artifacts. The drawback of the method is the long reconstruction time which makes it impossible to use in daily clinical practice, especially if the TV optimization problem has to be solved repeatedly to select a proper regularization parameter.

Materials and Methods  The goal of this work was to show that for the case of MR Angiography, TV filtering can be performed as a post-processing step, in contrast to the common approach of integrating TV penalties in the image reconstruction process. With this approach, it is possible to use TV algorithms with data fidelity terms in image space, which can be implemented very efficiently on graphic processing units (GPUs). The combination of a special radial sampling trajectory and a full 3D formulation of the TV minimization problem is crucial for the effectiveness of the artifact elimination process.

Results and Conclusion  The computation times of GPU-TV show that interactive elimination of undersampling artifacts is possible even for large volume data sets, in particular allowing the interactive determination of the regularization parameter. Results from phantom measurements and in vivo angiography data sets show that 3D TV, together with the proposed sampling trajectory, leads to pronounced improvements in image quality. However, while artifact removal was very efficient for angiography data sets in this work, it cannot be expected that the proposed method of TV post-processing will work for arbitrary types of scans.

Keywords  Angiography · Accelerated imaging · Radial sampling · Total variation · GPU computing

Introduction

The time window for data acquisition in contrast-enhanced MR angiography (CE-MRA) is limited due to the passage of the contrast agent. Additionally, a high spatial resolution is needed to visualize small vessels, while high temporal resolution is necessary to capture the dynamics of the contrast agent bolus. However, due to fundamental properties of MRI, there is always a trade-off between spatial and temporal resolution. As MRI data acquisition is sequential, it can be accelerated by reducing the number of measurement steps, but this leads to artifacts in the reconstructed images because the Nyquist criterion is violated. There are different strategies to mitigate these artifacts such as parallel imaging [1,2] or methods that exploit spatio-temporal correlations [3–5].
During the last years, reconstruction strategies were formulated that use tailored acquisition strategies like radial sampling or randomized 3D sampling patterns and include a priori knowledge about the imaged objects during the reconstruction process [6,7]. Examples for a priori knowledge are sparsity of the image, or total variation (TV)-based methods which assume that the image consists of areas that are piece-wise constant. Mathematically, these additional assumptions are introduced by reformulating image reconstruction as a constrained optimization problem. These algorithms allow the reconstruction of high-quality images from highly undersampled data sets, but at the price of long computation time. While this is not a severe limitation during research, it makes them currently impossible to use in daily clinical practice.

Hansen et al. [8] and Sorensen et al. [9] have shown recently that it is possible to use the massively parallel streaming processor architecture of modern graphic processing units (GPUs) to speed up image reconstruction for parallel imaging and for the reconstruction of non-Cartesian data. The goal of this work was to show that for the special case of CE-MR angiography data sets, which feature a high contrast to noise ratio, it is possible to use TV algorithms with data fidelity terms in image space, instead of integrating TV regularization in the image reconstruction process. For these algorithms, primal-dual-based TV formulations can be implemented very efficiently on the GPU, with computation times that make interactive elimination of undersampling artifacts possible. This is especially important for the interactive determination of the regularization parameter because a proper choice usually depends on patient-specific conditions like the geometry of the imaged anatomy, and therefore, predefined settings may not deliver optimal results. It is also shown in this paper that using a special type of radial sampling, 3D a priori information can be included in the regularization process, which significantly improves image quality. While this leads to an even higher computational complexity, it becomes feasible with the GPU implementation.

Theory

Undersampled radial imaging and total variation

It is well known [10] that in order to reconstruct an \( n \times n \) image matrix from a fully sampled radial data set, \( \frac{2\pi}{2} \cdot n \) radial projections have to be acquired. Reducing the number of projections accelerates data acquisition but leads to characteristic streaking artifacts in the reconstruction. It was shown by Block et al. [7] that these streaking artifacts can be reduced efficiently using a TV regularization in conjunction with a data fidelity term in k-space, a strategy that is comparable with compressed sensing approaches [6]. In contrast, our method is based on the original TV approach formulated in image space. TV-based minimization problems were originally designed for elimination of Gaussian noise and were first used by Rudin, Osher and Fatemi [11] in 1992. The denoising model using a TV regularization together with a \( L^2 \) data term in image space is, therefore, often referred to as the ROF model. It is defined as the following minimization problem:

\[
\min_u \left\{ \int_\Omega |\nabla u| \, dx + \frac{\lambda}{2} \int_\Omega (u - f)^2 \, dx \right\},
\]

where \( f \) is the original, corrupted image data. In the original work [11], the corruption is due to Gaussian noise, while in our case, it comes from streaking artifacts due to subsampling. The minimizer \( u \) is the reconstructed image, and \( \Omega \) is the image domain. The regularization parameter \( \lambda \) controls the balance between artifact removal—minimizing the TV penalty—and faithfulness to the original image—minimizing the data fidelity term—and so the proper choice of this parameter is a major challenge in any regularization-based reconstruction method. The \( L^2 \) norm of the data fidelity term makes the removal of structures contrast dependent: Low-contrast regions are removed, but strong contrast regions of the same size are kept. While this is desirable for high-contrast data such as CE-MR angiography, it is not appropriate for other data sets with low-contrast features. For such applications, it is possible to extend the proposed approach to include an \( L^1 \) data fidelity term, which is better suited due to its contrast invariance. The TV regularization has the advantage of removing noise while preserving sharp edges in the image. Therefore, vessels with their strong contrast-to-noise-ratio are preserved, while undersampling artifacts are efficiently removed. The main difference to the method in [7] is that we use a two-step procedure: First, a Fourier transform is applied to the MRI raw data, giving a reconstructed image containing aliasing artifacts. This image is then filtered through TV minimization. In contrast, Block et al. use a single-step procedure where the TV-based regularization is integrated in the image reconstruction process.

The minimization of the ROF model is a well-studied problem. As the functional defined in (1) is convex, the unique global minimizer can be calculated. In the original formulation, this was done using explicit time marching [11]. Other methods employ a linearization of the Euler–Lagrange equation [12,13]. Duality-based methods have shown greatly improved performance and were, among others, proposed in [14–16] and [17]. Recently, very fast primal-dual (PDU) approaches were proposed by Zhu et al. in [18] and [19], although they were already used for saddle point problems by Popov in [20]. In [21], a Split Bregman algorithm is used for the minimization. Discrete methods using graph cuts also can be used to solve the ROF model [22]. As shown in [23],