A computational study on general equilibrium pricing of derivative securities

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Abstract This paper analyses the accuracy of replicating portfolio methods in predicting asset prices. In a two-period, general equilibrium model with incomplete financial markets and heterogeneous agents, a computational study is conducted under various distributional assumptions. The focus is on the price of a call option on an underlying risky asset. There is evidence that the value of the (approximate) replicating portfolio is a good approximation for the general equilibrium price for CRRA preferences, but not for CARA preferences. Furthermore, there is strong evidence that the introduction of the call option reduces market incompleteness, but that the price of the underlying asset is unchanged. There is, however, inconclusive evidence on the welfare effects of the option.

Keywords Asset pricing · General equilibrium · Incomplete markets

JEL Classification Cation: D52 · G12

1 Introduction

Ever since the seminal contributions of Black and Scholes (1973), Merton (1973) and Harrison and Kreps (1979), there has evolved an entire industry concerned with asset
pricing based on martingale methods. These methods are based on the two fundamental theorems of finance, the first of which states that, in the absence of arbitrage opportunities, there exists a probability measure under which each asset’s price is the expectation of the future dividends. The second fundamental theorem then states that this measure is unique in complete markets. In a two-period economy with finitely many future states and one consumption good, the equivalent martingale measure can be interpreted as a vector of state prices, representing the price of consumption in each uncertain state in future.

In essence, financial markets are complete if the economy is isomorphic to an economy with a complete set of contingent contracts (a so-called Arrow–Debreu economy, see Debreu 1959, Chap. 7). Completeness allows the market to determine asset prices in such a way that all agents’ present value vectors are equalised. The present value vector of an agent lists her individual valuation of each future state (in terms of the consumption good) given all asset prices.

From a practical point of view the two fundamental theorems are extremely useful. If markets are complete then, in order to value a (new) financial asset, no knowledge whatsoever about preferences or initial endowments is needed. In complete markets, namely, any new asset is redundant and its no-arbitrage value can be computed as a linear combination of the prices of the existing assets. The weights are determined by the orthogonal projection of the dividend stream of the new asset on the market span and are called the replicating portfolio. The resulting price is called the replicating portfolio value (RPV).

In incomplete markets, however, the present value vectors are, generically, not equal in equilibrium. Their orthogonal projections on the market span, however, are. In other words, there are several valid vectors of state prices in equilibrium. This happens because the dimension of the subspace orthogonal to the market space is larger than one. This, in turn, implies that new assets can, generically, not be expected to be redundant. If a new asset is non-redundant no replicating portfolio exists. The best one can hope for is to find an approximate replicating portfolio via an orthogonal projection on the market space and, therefore, an approximate value for the new asset. This method has been advocated by, for example, Föllmer and Sondermann (1986).

There are two reasons why the RPV might lead to structural errors in predicting a new asset’s value in incomplete markets. Firstly, market incompleteness leads to non-existence of an exact replicating portfolio, as mentioned before. So, the portfolio used to compute the RPV is subject to approximation error. Secondly, the introduction of a new asset might change the prices of all other traded assets in equilibrium as well, due to the resulting change in the market span. This then leads to an error due to the fact that the “wrong” asset prices are used to determine the RPV. Indeed, there is

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1 In fact, if markets are sufficiently incomplete, no redundant options exist at all (see Baptista (2007) and references therein).
2 Whether or not actual financial markets are incomplete is open for debate. However, even if markets are, in fact, complete, asset valuation typically only uses a subset of all traded assets. That is, asset valuation then takes place in essentially incomplete markets (see Ross (2005) for an elaboration of this point).
3 This is reminiscent of the “Lucas-critique” in macroeconomics, see Lucas (1976).