BLOOD FLOW AND MACROMOLECULAR TRANSPORT IN CURVED BLOOD VESSELS *

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Abstract: A numerical analysis of the steady/pulsatile flow and macromolecular (such as LDL and Albumin) transport in curved blood vessels was carried out. The computational results predict that the vortex of the secondary flow is time-dependent in the aortic arch. The concentration of macromolecule concentrates at the region of sharp curve, and the wall concentration at the outer part is higher than that at the inner part. Atherosclerosis and thrombosis are prone to develop in such regions with sharp flow.

Key words: curved blood vessels; blood flow; macromolecular transport; atherosclerosis

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Introduction

The physiological anatomy indicates that atherosclerosis often occurs at complex flow regions (such as curved, stenosed or bifurcation regions) in the coronary artery, the carotid artery and the abdominal aortic\(^1\). It illuminates that the disease is at least partly related to the hemodynamic factors. On the other hand, atherosclerosis behaves as the local intimal accumulation of low-density lipoprotein (LDL), the increase of the fiberal tissue and the arterial stenosis, while the grease accumulation is related to the distribution of macromolecule (such as LDL) near the tube wall and its transport across wall. So, it is very important to study flow characters and macromolecular transport in complex blood vessels (curved, stenosed and bifurcation blood vessels), and it could be helpful to discover the mechanism of atherogenesis.

A number of studies about flow in curved blood vessels have been carried out. In 1927, Dean\(^2\) first introduced the model of uniform curvature and round section for the curved blood vessel and discovered the secondary flow in it. Shalcheraghi et al.\(^3\)–\(^5\) studied pulsatile flow in the human aortic arch and its three main branches numerically, revealed a wide variety of interesting phenomena including complex vortex structures. Torii et al.\(^6\) numerically simulated the blood flow in the cerebral artery using CT and obtained the patterns of wall shear stress and velocity of secondary flow. Moore et al.\(^7\)–\(^8\) investigated blood flow in coronary arteries and made clear the importance of varying curvature on flow patterns and wall shear stress. Qiao et al.\(^9\) made a finite element numerical simulation of developing blood flow in curved arteries. About mass transport, Rappitsch et al.\(^10\)–\(^12\) studied mass transport in axisymmetric stenosed arteries and a separated flow region of an sudden expansion. We have also carried out a series

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of studies about mass transport in coronary arterial stenosis, small curvature ratio tubes and T-bifurcation blood vessels\textsuperscript{[13–16]}. But work on the macromolecular transport in curved blood vessels is rare.

In the present study, the steady/pulsatile flow and macromolecular (such as LDL and Albumin) transport in curved blood vessels were analyzed numerically. The hemodynamic factors such as axial velocity, secondary flow and wall shear stress were obtained as well as concentration distributions of Albumin and LDL. And possible relatives of these factors with atherosclerosis were discussed.

1 Mathematical Model

1.1 Basic assumptions
i) Rigid arterial wall; ii) Incompressible homogeneous Newtonian fluid; iii) Under normal physiological condition, water and Albumin could across the blood wall, but LDL is much larger, it could hardly go through the blood wall, so we assumed LDL could not transport across wall.

1.2 Coordinates and grid systems

Supposing $a$ is the radius of curved tube, $R$ and $L_1$ are curvature radius and length of the straight tube for the aortic arch model. Taking the curving center of the curved tube as origin $O$, the vertical axis of the straight tube as axis $y$, the symmetry plane as plane $y$-$z$, and the axis vertical to the symmetry plane as axis $x$, the right hand coordinates $(x, y, z)$ were shown in Fig.1 structured grids were generated based on the method in Ref.[17] as shown in Fig.2.

\[ \text{Fig.1 Coordinates} \quad \text{Fig.2 Grid systems} \]

1.3 Governing equations and boundary and initial conditions

Using the method of artificial compressibility to normalize the N-S equations, $L(L = 2a$, diameter of the tube) is the reference length, velocity $u$, time $t$ and pressure $p$ are normalized by the inlet velocity $U_0$, $L$, and $p_0 = \rho U_0^2$, respectively. A fictitious time derivative of pressure is added to the mass conservation equation in the curvilinear coordinates system $(\xi, \eta, \zeta)$. In the present work, a fictitious time derivative $\frac{\partial}{\partial \tau}$ is also added to momentum equations. For steady flow the term $\frac{\partial}{\partial t}$ is zero. The complete Navier-Stokes equations are written in artificial compressible conservative form\textsuperscript{[18]} as

\[ I_m \frac{\partial q}{\partial \tau} + \frac{\partial q}{\partial \tau} + \frac{\partial (E - E_v)}{\partial \xi} + \frac{\partial (F - F_v)}{\partial \eta} + \frac{\partial (G - G_v)}{\partial \zeta} = 0, \quad (1) \]

where

\[ I_m = \text{diag}(0, 1, 1, 1), \quad (2) \]