PSE as applied to problems of secondary instability in supersonic boundary layers *

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Abstract Parabolized stability equations (PSE) approach is used to investigate problems of secondary instability in supersonic boundary layers. The results show that the mechanism of secondary instability does work, whether the 2-D fundamental disturbance is of the first mode or second mode T-S wave. The variation of the growth rates of the 3-D sub-harmonic wave against its span-wise wave number and the amplitude of the 2-D fundamental wave is found to be similar to those found in incompressible boundary layers. But even as the amplitude of the 2-D wave is as large as the order 2%, the maximum growth rate of the 3-D sub-harmonic is still much smaller than the growth rate of the most unstable second mode 2-D T-S wave. Consequently, secondary instability is unlikely the main cause leading to transition in supersonic boundary layers.

Key words parabolized stability equations, secondary instability, fundamental disturbances, sub-harmonic waves

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Introduction

The problem of laminar-turbulent transition is of great interest. For incompressible flows, transition may start from the linear amplification of a small 2-D disturbance. When the amplitude of 2-D disturbance becomes larger, 3-D disturbances may be generated due to nonlinear effect and start to play an important role in transition, leading to turbulent flows finally. One of the possible mechanisms for the generation of 3-D disturbances is the secondary instability mechanism, as introduced by Herbert\(^{[1–3]}\). This mechanism has been confirmed by experiments\(^{[4]}\), manifested in the appearance of A shaped structures shown by flow visualization. The variation of the growth rate of the 3-D sub-harmonic wave against its span-wise wave number has also been measured\(^{[5–7]}\).

The problem concerned in this paper is, whether the same mechanism works for supersonic boundary layers. So far, there were only a few works in this regard. One was the experimental

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work of Maslov, in which sub-harmonic has been detected in the transition, but one can hardly confirm if it is due to secondary instability, because it was only signals detected by hot wire anemometer. The other was the numerical study of Dong & Zhou\[8\]. They carried out direct numerical simulations (DNS) so that secondary instability mechanism was confirmed and the variations of the growth rates of sub-harmonic waves against its span-wise wave number and the amplitude of the 2-D fundamental were discovered. However, DNS is time consuming, so that the number of cases investigated is surely very limited. As a result, the threshold amplitude of the fundamental waves for producing unstable sub-harmonic waves was not given by Dong & Zhou.

In the past few years, PSE approach for the compressible flows has been developed\[9–11\], as its computational time required is significantly less. Zhang & Zhou\[12\] verified the effectiveness of PSE method for compressible boundary layers and found that results from PSE agreed quite well with those from DNS. Consequently, PSE could be a reliable tool to investigate the evolution of disturbances in supersonic boundary layers.

In this paper, PSE method is used to investigate the secondary instability in supersonic boundary layers.

1 Governing equations

Starting from the full Navier-Stokes (N-S) equations, the full disturbance equations can be obtained. Then, stability equations were derived, with the characteristics of disturbances being taken into account. Finally, equations are parabolized to become PSE, under the assumption that the growth of the boundary layer thickness is slow in the stream-wise direction.

With the superscript * denoting the dimensional quantities, the full N-S equations in 3-D Cartesian coordinates are

\[
\frac{\partial \rho^*}{\partial t^*} + \nabla \cdot (\rho^* \mathbf{V}^*) = 0,
\]

\[
\rho^* \left[ \frac{\partial \mathbf{V}^*}{\partial t^*} + (\mathbf{V}^* \cdot \nabla) \mathbf{V}^* \right] = -\nabla p^* - \nabla \times [\mu^* (\nabla \times \mathbf{V}^*)] + \nabla [(\lambda^* + 2\mu^*) \nabla \cdot \mathbf{V}^*],
\]

\[
\rho^* c_p^* \left[ \frac{\partial T^*}{\partial t^*} + (\mathbf{V}^* \cdot \nabla) T^* \right] = \nabla \cdot (k^* \nabla T^*) + \frac{\partial p^*}{\partial t} + (\mathbf{V}^* \cdot \nabla) p^* + \Phi^*,
\]

\[
p^* = \rho^* R^* T^*,
\]

where \( \mathbf{V}^* = (u^*, v^*, w^*)^T \) is the velocity vector, \( t^* \) the time, \( \rho^* \) the density, \( p^* \) the pressure, \( T^* \) the temperature, \( c_p^* \) the specific heat, \( k^* \) the thermal conductivity, \( \mu^* \) the first coefficient of viscosity, \( \lambda^* \) the second coefficient of viscosity, and \( R^* \) the gas constant, \( \Phi^* \) the viscous dissipation function, expressed as

\[
\Phi^* = 2\mu^* \mathbf{S}^* : \mathbf{S}^* + \lambda^* (\nabla \cdot \mathbf{V}^*)^2,
\]

where \( \mathbf{S}^* \) is the strain rate tensor.

To make the equations non-dimensional, \( l_0^* = \sqrt{\gamma_c^* x_0^* / u_e^*} \) is used as the reference length scale, where \( x_0^* \) is the distance from the leading edge to the entrance of the computational domain, \( \gamma_c^* \) the kinematic viscosity, \( u_e^* \) the free stream velocity, and the subscript "c" implies value taken at the outer edge of the boundary layer. Other reference quantities are the free stream velocity \( u_e^* \) and free stream temperature \( T_e^* \). The resulting non-dimensional distance from the leading edge to the entrance of the computational domain is \( x_0 = x_0^* / l_0^* = \sqrt{u_e^* x_0^*/\gamma_c^*} \), the corresponding Reynolds number is \( Re_0 = u_e^* l_0^*/\gamma_c^* = \sqrt{u_e^* x_0^*/\gamma_c^*} = x_0 \). The resulting non-dimensional N-S equations are then obtained, with these reference quantities being used.