Unsteady magnetohydrodynamic stagnation point flow — closed-form analytical solutions

T. G. FANG\textsuperscript{1,\dagger}, F. J. WANG\textsuperscript{1}, Bo GAO\textsuperscript{2}

1. Mechanical and Aerospace Engineering Department, North Carolina State University,
Raleigh, NC 27695, U. S. A.;
2. School of Energy & Power Engineering, Jiangsu University, Zhenjiang 212013,
Jiangsu Province, China
(Received Jul. 2, 2018 / Revised Sept. 11, 2018)

Abstract  This paper investigates the unsteady stagnation point flow and heat transfer of magnetohydrodynamic (MHD) fluids over a moving permeable flat surface. The unsteady Navier-Stokes (NS) equations are transformed into a similarity nonlinear ordinary differential equation, and a closed form solution is obtained for the unsteadiness parameter of 2. The boundary layer energy equation is transformed into a similarity equation, and is solved for a constant wall temperature and a time-dependent uniform wall heat flux case. The solution domain, velocity, and temperature profiles are calculated for different combinations of parameters including the Prandtl number, mass transfer parameter, wall moving parameter, and magnetic parameter. Two solution branches are obtained for certain combinations of the controlling parameters, and a stability analysis demonstrates that the lower solution branch is not stable. The present solutions provide an exact solution to the entire unsteady MHD NS equations, which can be used for validating the numerical code of computational fluid dynamics.

Key words  unsteady stagnation point flow, stretching/shrinking sheet, magnetohydrodynamic (MHD), Navier-Stokes (NS) equation

Chinese Library Classification  O361
2010 Mathematics Subject Classification  76W05

1 Introduction

Stagnation point flows occur in many practical situations when there is a flow impinging on an object. Yang\textsuperscript{[1]} was the first who investigated the unsteady stagnation point flow over a flat plate using a similarity transformation technique. He analyzed both the momentum and thermal boundary layers, and his solution is an exact solution of the whole unsteady Navier-Stokes (NS) equations. Yang’s work was further generalized by Williams III\textsuperscript{[2]} to the stagnation point flow in an axisymmetric setting and by Jankowski and Gersting\textsuperscript{[3]} to a general three-dimensional configuration. Teipel\textsuperscript{[4]} studied the heat transfer problem of the three-dimensional

---


\dagger Corresponding author, E-mail: tfang2@ncsu.edu

©Shanghai University and Springer-Verlag GmbH Germany, part of Springer Nature 2019
unsteady stagnation point flow and analyzed the heat transfer characteristics under different unsteadiness parameters. Wang\cite{5} further extended the problem to the unsteady stagnation point flow considering the impinging directions. Rajappa\cite{6} investigated the mass transpiration effects for Yang’s problem. Burde\cite{7} studied the unsteady NS equations and proposed some new solutions to the unsteady stagnation point flow. Ludlow et al.\cite{8} further analyzed the unsteady boundary layer equations and obtained new solutions. Ma and Hui\cite{9} performed an analysis on the two-dimensional boundary layer equations and proposed a new type of unsteady separated stagnation point flow.

Until recent years, there are still many active research activities in the literature in studying the unsteady stagnation point problem. Takhar et al.\cite{10} investigated the unsteady axisymmetric stagnation point flow over a circular cylinder. Unsteady mixed convection near the three-dimensional stagnation point region was studied by Eswara and Nath\cite{11}, and the effects of large mass injection were analyzed. The flow and heat transfer problems of a magnetohydrodynamic (MHD) fluid in the stagnation region of a three-dimensional body were investigated by Kumari and Nath\cite{12}. Unsteady three-dimensional viscoelastic fluid flow near the stagnation point was analyzed by Seshadri et al.\cite{13}. Xu et al.\cite{14} employed an analytical method to obtain series solutions to the unsteady stagnation point flow of non-Newtonian flows. Unsteady stagnation point flow over a stretching sheet was investigated by Nazar et al.\cite{15}, and this problem was extended to a second grade fluid by Baris and Doku\cite{16}. The mass transfer effects on Yang’s problem were studied in detail by Fang et al.\cite{17}, and multiple solution branches were found for both momentum and thermal boundary layers. The unsteady stagnation point flow over a plate moving along the flow impingement direction was studied by Zhong and Fang\cite{18} for both two-dimensional and axisymmetric cases. It should be noted that in all these works, the solutions to the similarity equations were not given in a closed form. Magyari and Weidman\cite{19} provided a closed form solution for a special condition of Yang’s problem. Fang and Zhong\cite{20} derived a closed form solution to Yang’s problem including mass transpiration and wall movement.

Moreover, the unsteady stagnation point flow of MHD fluids also received much attention in the literature due to its important engineering applications. A few recent examples are discussed here. Soid et al.\cite{21} investigated the MHD stagnation point flow over a stretching/shrinking sheet using numerical techniques. Multiple solutions were found in the work depending on the wall movement parameter. Chen et al.\cite{22} extended the unsteady MHD stagnation point flow over a shrinking sheet by including thermal radiation and slip effects. Turkylmazoglu et al.\cite{23} generalized the unsteady MHD flow to the rear stagnation point configuration over off-centred deformable surfaces. Zaib et al.\cite{24} studied the heat and mass transfer of an unsteady MHD stagnation point flow of a nanofluid by considering the effects of thermophoresis. More related works on flows over moving boundaries of different types of fluids and the MHD flows for different flow configurations can be found in Refs.\cite{25}–\cite{30}. It should be pointed out that for the forward unsteady MHD stagnation point flow, there is no solution in a closed form reported in the literature. Therefore, in this paper we will further extend the work in Ref.\cite{20} to include the hydromagnetic effects on the general unsteady stagnation point flow passing a moving surface. The energy equation will also be investigated for a constant wall temperature case and a uniform wall heat flux case.

2 Theoretical derivation

2.1 Momentum and energy equations

We consider a two-dimensional laminar viscous incompressible stagnation point flow of MHD fluids over a flat plate with an unsteady free stream velocity \( U_\infty = U_0 x (1 - \gamma t)^{-1} \), where \( U_0 \) and \( \gamma \) are constants. The \( x \)-axis points to the free stream direction. The \( y \)-axis is perpendicular to the \( x \)-axis. The wall moves in the \( x \)-direction with a velocity of \( U_w = \lambda U_0 x (1 - \gamma t)^{-1} \) and \( \lambda = \frac{U_w}{U_\infty} \). A mass transfer velocity in the \( y \)-direction exists at the wall as \( V_w(x, t) \), which is to be