A COMPARISON OF SOLVERS FOR LINEAR
COMPLEMENTARITY PROBLEMS ARISING FROM
LARGE-SCALE MASONRY STRUCTURES*

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(Received September 26, 2005)

Abstract. We compare the numerical performance of several methods for solving the
discrete contact problem arising from the finite element discretisation of elastic systems
with numerous contact points. The problem is formulated as a variational inequality and
discretised using piecewise quadratic finite elements on a triangulation of the domain. At the
discrete level, the variational inequality is reformulated as a classical linear complementarity
system. We compare several state-of-art algorithms that have been advocated for such
problems. Computational tests illustrate the use of these methods for a large collection
of elastic bodies, such as a simplified bidimensional wall made of bricks or stone blocks,
deformed under volume and surface forces.

Keywords: linear elasticity, equilibrium problems, variational inequality, complementar-
ity problems, masonry structures

MSC 2000: 74B10, 74G15, 49J40, 90C33, 74L99

1. Introduction

An important problem arising in practical engineering applications involves a col-
lection of linearly elastic bodies that are deformed due to volume and surface forces,
but cannot penetrate each other. The work presented in this paper is motivated by
our interest in masonry structures. We assume that they can be modelled satisfacto-
ribly as a linear elasticity system assembled from a large number of elastic components
situated at nonnegative distance from one another. Our present objective is to com-

*This work was supported by the Engineering and Physical Science Research Council of
Great Britain under grant GR/S35101, and the first author was supported by a fellowship
from the Royal Society of Edinburgh.
pare several state-of-art algorithms that have been advocated for the solution of the linear complementarity problem that arise when such problems are discretised.

In Section 2, we describe the model problem in terms of classical partial differential equations of linear elasticity with contact conditions. The problem is formulated as a variational inequality and discretised using piecewise quadratic finite elements on a triangulation of the domain. The treatment of variational inequalities and their applications in continuum mechanics is discussed, for example, in Fichera [4], Duvaut & Lions [3], Glowinski et al. [6], Hlaváček et al. [11], Kikuchi & Oden [13]. In Section 3, at the discrete level, the variational inequality is reformulated as a classical linear complementarity system. In Section 4, we discuss several iterative solvers for the discrete constrained system. The solvers we consider are: the successive over-relaxation with projection, cf. e.g. Glowinski et al. [6], the linear least squares with nonnegativity constraints, cf. Lawson & Hanson [14], the primal-dual active-set method, cf. Hintermüller et al. [9], the primal-dual predictor-corrector method, cf. e.g. Wright [18], and the principal pivoting simplex method, cf. Graves [7]. In Section 5, numerical experiments are presented to illustrate the use of these solvers for a large collection of elastic bodies, such as a simplified bidimensional wall made of bricks or stone blocks, deformed under volume and surface forces. Concluding remarks are addressed in Section 6.

2. Formulation and discretisation of the model problem

We introduce the model contact problem in both the strong and weak forms and discuss the finite element approximation of the problem expressed as a variational inequality. Both the primal formulation of the problem (i.e. in terms of displacements only) and the primal-dual formulation (i.e. in terms of displacements and stresses) will be needed in view of the fact that the different solvers we consider are sometimes viewed more naturally in terms of the primal or dual problem.

The mathematical model. We consider an elastic system consisting of a finite, but possibly large, number of elastic bodies situated at nonnegative distance from one another (Fig. 1). Each body occupies a Lipschitz domain $\Omega^k \subset \mathbb{R}^d$, $d = 2$ or 3, $k = 1, \ldots, n_b$, and the domain occupied by the overall system is defined as $\Omega = \Omega^1 \cup \ldots \cup \Omega^{n_b}$. Let $\partial \Omega = \partial \Omega^1 \cup \ldots \cup \partial \Omega^{n_b}$ represent the global boundary of $\Omega$. We denote by $\Gamma_C$ the potential contact surface between the elastic bodies, and by $\Gamma_D = \partial \Omega \setminus \Gamma_C$ the exterior boundary of the overall system.

Let $u(x) = (u_1(x), \ldots, u_d(x))$, $x \in \Omega$, denote the vector field of displacements of the elastic system, and let $e_{kl} (k, l = 1, \ldots, d)$ represent the corresponding linearised