COVARIANT FORMULATION OF THE DYNAMICAL EQUATIONS OF QUANTUM VORTICES IN TYPE II SUPERCONDUCTORS

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We have derived the closed system of covariant equations which describe the motion of quantum vortices regarded as a two-dimensional polarized liquid. We have obtained the covariant expressions of the forces acting on the vortices; from the equilibrium condition of these forces we have deduced the equation satisfied by the velocity field of the fluid. It is shown that this velocity field depends on the friction coefficient, the density of vortices and the superconducting current. From this closed system of equations we derived the relaxation equation when a variable magnetic field is applied.

Key words: superconductors: dynamical equations

1. Introduction

It is a well-known fact that quantum vortices are generated in type II superconductors ($\lambda/\xi > 1/\sqrt{2}$) when the intensity of the applied magnetic field $H$ exceeds that of the critical field $H_c$ [1]. Such a system of vortices is generated in the core of neutron stars where protons are superconducting and the ratio $\lambda/\xi > 1$; vortices are also generated in a rotating neutron superfluid [2,3]. The dynamical equations describing the motions of vortices in rotating superfluids and type II superconductors, within the framework of the Newtonian theory, have been examined in [3,4]. The relativistic generalization of the corresponding equations for a rotating superfluid has been presented in [5]. The purpose of this paper is to derive the equations of motion of vortices for type II superconductors in general relativity. We shall see that these equations connect the magnetic field to the density of vortices and permit the study of: 1) relaxation process of vortices in a time-varying magnetic field, 2) the behavior of magnetic fields in the core of neutron stars. In Sec. 2 we derive the dynamical equations of quantum vortices. Adopting Synge's approach of electromagnetism in general relativity the relation between magnetic flux and number of vortices is given. In Sec. 3, we present a covariant formulation of the forces

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acting on vortices, i.e., the friction force and the force due to the supercurrent. In Sec. 4, from the equilibrium condition of the acting forces the dependence of the vortex velocity field on the supercurrent is derived. In Sec. 5 we derive the relaxation equation for the superconducting current and obtain the formula for the relaxation time. In Sec. 6 we summarize our results and note that a detailed investigation of the relaxation process requires specification of the superconducting medium under consideration. In the Appendix, following London, we briefly sketch the derivation of Eq.(10) by considering the motion of a superparticle subject only to the action of the Lorentz and Magnus forces.

2. The dynamical equations of quantum vortices in type II superconductors

Consider a static universe with metric of the form

\[ ds^2 = g_{ij} dx^i dx^j + g_{44} (dx^4)^2 , \]  

(1)

where the \( g \)'s are independent of the time coordinate \( x^4 \). In this universe we have a type II superconductor with world lines of the normal part along the \( x^4 \)-lines; consequently

\[ u^i (n) = 0, \quad g_{44} (u^4 (n))^2 = -1, \quad u^4 (n) = \frac{1}{\sqrt{-g_{44}}} . \]  

(2)

As is well known, when the intensity of the applied magnetic field \( H \) is less than the critical value \( H_{c1} \) for the creation of quantum vortices, the equations which relate the supercurrent to the electromagnetic field are the London equations that, in covariant form, read

\[ \nabla \mu \nu M = s_{\mu \nu} = 0 , \]  

(3)

where \( \nabla \) is the operator of covariant derivation and

\[ M_\nu = \frac{me}{\epsilon^2 n(s)} j_\nu + A_\nu , \]  

(4)

with \( e (e < 0) \), \( m \), and \( n(s) \) denoting respectively the charge, mass, and number density of superelectrons. The quantity \( j_\nu \) is the 4-current defined by

\[ j_\nu = e n(s) u_\nu (s) , \]  

(5)

and \( A_\nu \) the 4-potential connected to the electromagnetic field tensor by

\[ F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \]  

(6)

A consequence of Eq.(3) is the vanishing of the flux of magnetic field across a spacelike region \( V_2 \). Indeed, let \( t_{(1)}^\alpha \), \( t_{(2)}^\alpha \), \( t_{(3)}^\alpha \), and \( t_{(4)}^\alpha \) be an orthogonal tetrad of unit vectors defined at each point of the 2-dimensional spacelike region \( V_2 \) bounded by a closed curve \( V_1 \). Our \( t \)-tetrad has the same orientation as that of the parametric lines of the cylindrical coordinates \( (x^1, x^2, x^3, x^4) = (r, \phi, z, ct) \). The quantities \( t_{(1)}^\alpha \) and \( t_{(2)}^\alpha \) are chosen tangent to \( V_2 \) and are oriented along the radial and azimuthal parametric lines, \( t_{(3)}^\alpha \) and \( t_{(4)}^\alpha \) are of course normal to \( V_2 \) and tangent, respectively, to the \( x^3 = z \)