SUBOPTIMAL PROPERTIES OF AN IDEAL CASCADE WITH SYMMETRIC STEPS

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A computational experiment is used as a basis for comparing an ideal cascade with symmetric steps and a cascade optimized with respect to the total feed flux of the steps and identical external parameters for both cascades. It is shown that for large separation effects at the steps and a high degree of separation the parameters of an ideal cascade differ from an optimal cascade and its total feed flux at the steps is higher. It is established that the separation coefficients of the steps of an optimal cascade with respect to the enriched and depleted fractions are regular functions of the feed concentration at the steps and allow for higher specific separation power of the steps.

Several questions which are of great practical value in the theory of the separation of binary mixtures in cascades are still unclear. These include the determination of the separation potential and the separation power and the associated separation efficiency criteria. It is thought that this is due mainly to the difficulties in determining the best cascade for the general case of arbitrary separation effects at the steps [1].

The concept of an ideal cascade with symmetric steps was introduced in the monograph by K. Cohen [2], which is devoted to the separation of a binary mixture of isotopes. This cascade is characterized by equal separation coefficients of a step with respect to the enriched and depleted fractions and the absence of mixing of flows with different feed concentration at the steps. For weak enrichment, it corresponds to minimizing the total flux. Computational investigations have shown that an ideal cascade with arbitrary and asymmetric enrichment is inefficient [1, 3]. Its total flux, which can be compared with the number of separating elements, is much higher than for a cascade which is optimal with respect to this criterion.

The present work is a continuation of the investigations performed in [4]. The objective is to compare the optimal and ideal cascades with symmetric steps with large separation effects. Special calculations were performed to solve this problem. It is shown that for a high degree of separation and a large number of steps the ideal cascade is suboptimal. Its total flux is higher than for an optimal cascade. To explain the results obtained, the conditions for a maximum of the specific separation powers of the steps, determined according to the separation potential of P. Dirac, are analyzed.

Special Features of the Problem. A comparative analysis of the efficiency of the cascades was performed for the example of a counterflow symmetric scheme with three external flows (Fig. 1). For a binary mixture, the scheme is characterized by six external parameters: \( F, P, W \) – feed, product, and waste fluxes and \( C_F, C_P, C_W \) – the corresponding concentration of the valuable (light) component. Four of these parameters are independent. The cascade consists of \( n \) steps connected in series. Each step has similar parameters. In the notation adopted in Fig. 1, \( L_i, L_i', L_i'' \) are the feed, product, and waste fluxes at the \( i \)th step, \( i = 1, n \); \( C_i, C_i', C_i'' \) are the corresponding concentrations of the valuable component. The six \( n \) parameters listed are related by the balance equations, the equations relating the fluxes between the steps, and the separation characteristics of the steps. If the total coefficients of separation of the steps \( q_i (i = 1, n) \) are assumed to be known, then the number of unknown fluxes and the concentration of the steps comprise \( n \) parameters.
The independent parameters of a cascade can be chosen on the basis of the efficiency criterion adopted. To determine the total number of independent external and internal parameters, it should be kept in mind that the balance relations for the external parameters are always satisfied if balance holds at all steps. In addition, the external and internal parameters are related by the boundary conditions. Hence it follows that for fixed \( n \) and \( f \) (the number of the external-feed step) the number of independent parameters of a binary cascade is \( n + 2 \).

An ideal cascade can be constructed by setting the feed concentration \( C_F \), one of the external flows, and by satisfying the nonmixing conditions:

\[
C_{i}^{'} = C_{i+1}^{''}, \quad i = \frac{2}{n-1}, \quad (1)
\]

\[
C_f = C_F. \quad (2)
\]

For two prescribed external parameters and the conditions of (1) and (2), there remains one free parameter which can determine the type of ideal cascade.

The (1) conditions mean that the separation coefficients with respect to the enriched \( \alpha_i \) and depleted \( \beta_{i+1} \) fractions of two neighboring steps are equal, i.e., \( \alpha_i = \beta_{i+1}, \quad i = 1, n-1 \). For steps with the same total separation coefficients \( q_i = q \), choosing symmetric operation of one step, for example, \( \alpha_1 = \beta_1 \), results in a symmetric ideal cascade characterized by equal separation coefficients \( \alpha \) and \( \beta \) for all steps: \( \alpha = \beta = \sqrt{q.} \) This cascade can be calculated using known relations with the external feed step [5]. The special feature of this cascade is that the product concentration \( C_P \) and the waste concentration \( C_W \) are calculated according to the fixed feed concentration \( C_F \), the number of steps \( n \), the number of the external feed step \( f \), and the separation coefficient \( \alpha \). In contrast, an optimal cascade can be constructed for any prescribed product concentration \( C_P \) and waste concentration \( C_W \) [3]. It is a mixing cascade. The remaining \( n - 2 \) free internal parameters are optimized according to the criterion for minimum total flux, i.e., \( \sum_{i=1}^{n} L_i \to \min \). In addition, the cascade has additional degrees of freedom, associated with the possibility of varying \( n \) and \( f \). This makes it possible to determine the best parameters from the standpoint of the total flux as compared with the ideal cascade.

Using these features as a basis, cascades with identical external parameters, determined as a result of a calculation of the ideal cascade, were compared. The optimal cascade was calculated using the optimization method of [3] together with the Hooke–Jeeves numerical method. The method developed made it possible to calculate cascade fluxes with an error down to \( 10^{-15} \).

**Comparison of Optimal and Ideal Cascades.** The calculations showed that for a large total separation coefficients \( q \) of the steps and high degrees of separation \( C_P/C_W \) an ideal cascade with symmetric steps does not provide the minimum total flux. The difference with respect to the optimal cascade is easily seen for relatively low values of the separation coefficients \( q = 5 \). In this case, if the product flux concentration \( C_P \) is several percent and the waste flux concentration \( C_W = 10^{-5}\% \), then the total flux of an optimal cascade is less than one tenth of a percent. For larger separation coefficients, up to \( q = 20 \), the difference increases to several percent.

Computational results illustrating the properties noted above are presented as an example in Table 1. For all cascades, the product flux \( P = 1 \) g/sec. Cases of high enrichment characterized by a low waste concentration \( C_W \) are shown.