COMPUTED TRITIUM CONCENTRATION
IN A HEAVY-WATER REACTOR

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The solutions of a system of equations governing the accumulation of tritium and helium in the heavy water of a reactor are presented for arbitrary initial conditions. The solutions are presented as functions of heavy-water exchange and helium extraction. The concentration change is illustrated for typical values of the thermal-neutron flux density.

Tritium is produced in heavy water as a result of the reaction $\text{D}(n, \gamma)\text{T}$ and is converted into $^3\text{He}$ as a result of $\beta$ decay. The helium produced in the reaction $^3\text{He}(n, p)\text{T}$ is reconverted into tritium. The equations for the tritium $N_T$ and helium $N_{He}$ concentrations for constant volume of the heavy-water loop have the following form:

\[
\frac{dN_T}{dt} = \sigma_D \Phi N_D - (\lambda + \gamma)N_T + \sigma_{He} \Phi N_{He} + \gamma N_T^0; \\
\frac{dN_{He}}{dt} = \lambda N_T - (\sigma_{He} \Phi + \gamma + \beta)N_{He},
\]

where $\sigma_D \Phi$ and $\sigma_{He} \Phi$ are the rates of neutron capture by the deuterium and helium, respectively, averaged over the entire volume of the heavy-water loop taking account of the time coefficient of power utilization; $N_D$ is the density of deuterium nuclei in heavy water; $\gamma$ is the rate of exchange of heavy water in the loop for heavy water with tritium concentration $N_T^0$ and without helium (detrinitization rate); $\beta$ is the rate of helium extraction from the heavy water. In Eqs. (1), the burnup of tritium and deuterium is neglected because the contribution of these processes is small.

For values of $\Phi$, $\beta$, and $\gamma$ which remain constant in time, the solution of system (1) has the following form:

\[
N_{He}(t) = N_{He}^{as} + \left\{[N_{He}^{as} - N_{He}(0)](\eta_1 \exp(\eta_1 t) - \eta_2 \exp(\eta_2 t)) + \right.\\
\left. + \left\{[\sigma_{He} \Phi + \gamma + \beta]N_{He}(0) - \lambda N_T(0)\right\}(\exp(\eta_1 t) - \exp(\eta_2 t))\right\}/(\eta_2 - \eta_1); \\
N_T(t) = N_T^{as} + \left\{[N_T^{as} - N_T(0)](\eta_1 \exp(\eta_1 t) - \eta_2 \exp(\eta_2 t)) + \right.\\
\left. + \left\{[\sigma_D \Phi N_D + \gamma N_T^0 - (\lambda + \gamma)N_T(0) + \sigma_{He} \Phi N_{He}(0)](\exp(\eta_1 t) - \exp(\eta_2 t))\right\}/(\eta_2 - \eta_1),
\]

where

\[
\eta_{1,2} = -0.5(\sigma_{He} \Phi + 2\gamma + \beta + \lambda) \pm 0.5\sqrt{(\sigma_{He} \Phi + \beta - \lambda)^2 + 4\sigma_{He} \Phi \lambda};
\]
He(0) and N_T(0) are the initial helium and tritium concentrations at t = 0 and N^{as}_{He} and N^{as}_{T} are their asymptotic values.

If there is no heavy-water exchange or helium extraction (γ = β = 0), then for prolonged operation, i.e., for t >> (σ_{He} Φ + λ), the helium and tritium concentrations increase linearly with time:

\[ N^{as}_{He} = \frac{λ(σ_{D} Φ N_{D} + γ N_{T}^{0})}{σ_{He} Φ γ + (γ + β)(γ + λ)}; \]  
\[ N^{as}_{T} = \frac{(σ_{He} Φ + γ + β)(σ_{D} Φ N_{D} + γ N_{T}^{0})}{σ_{He} Φ γ + (γ + β)(γ + λ)}, \]

N_{He}(0) and N_{T}(0) are the initial helium and tritium concentrations at t = 0 and N^{as}_{He} and N^{as}_{T} are their asymptotic values.

If there is no heavy-water exchange or helium extraction (γ = β = 0), then for prolonged operation, i.e., for t >> (σ_{He} Φ + λ), the helium and tritium concentrations increase linearly with time:

\[ N_{He} \sim \frac{σ_{D} Φ N_{D} λ}{σ_{He} Φ} t; \]  
\[ N_{T} \sim \frac{σ_{D} Φ N_{D} σ_{He} Φ}{σ_{He} Φ + λ} t. \]

It follows from Eq. (6) that the rate of growth of the concentration of helium nuclei in this case at high fluxes (σ_{He} Φ >> λ) will be independent of the neutron flux density. Relation (7) under the same conditions shows that the rate of growth of the tritium concentration is proportional to the neutron flux density.

In reality there is always some leakage of heavy water, and γ ≠ 0. If γ or β is different from zero, then in time N_{He} and N_{T} will reach the asymptotic values N^{as}_{He} and N^{as}_{T}. It follows from relations (4) and (5) that for nonzero γ and high neutron flux density the helium concentration is independent of and the tritium concentration is proportional to the neutron flux density.

When the reactor is shut down after operating for time t_0, solutions (2) and (3) assume the following form:

\[ N_{T}(t) = N_{T}(t_0) \exp(S_1(t-t_0)) - γ N_{T}^{0}[1 - \exp(S_1(t-t_0))] / S_1; \]