EFFECTS DUE TO MULTIPLE SCATTERING
OF FISSION FRAGMENTS IN LAYERS
OF FISSIONING MATERIAL

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Computer simulation with the SRIM program is used to investigate the passage of fragments from $^{235}\text{U}$ fission by thermal neutrons through 50–400 µg/cm$^2$ thick uranium layers. The calculations are performed in the two-fragment approximation: light $^{97}\text{Mo}$ and heavy $^{139}\text{Ba}$. The energy spectra of the fission fragments emanating from the layer are calculated and the effects due to multiple scattering of the fragments in thin layers are estimated.

The multiple scattering of fission fragments in layers of a fissioning material and the associated backscattering from the substrates (which are often thick) on which the fissioning layer is deposited changes the energy and angular distribution of the fragments emanating from the layer and the total number of fragments [1]. In fission detectors, these processes influence the detection efficiency $\varepsilon$ of the fission events [2, 3], which is defined as

$$\varepsilon = \varepsilon_0 P,$$

where $\varepsilon_0$ is the probability that fragments leave the fissioning layer and $P$ is the probability that fragments are detected, which depends on the characteristics of the detector and electronic apparatus. In experiments measuring the neutron fission cross sections for nuclei such as $^{235}\text{U}$, $^{233}\text{U}$, and $^{239}\text{Pu}$, it is necessary to know the value of $\varepsilon_0$ to better than 0.5%. In a fragment stopping model based on rectilinear trajectories and continuous retardation (neglecting multiple scattering), the quantity $\varepsilon_0$ for an infinite flat layer of thickness $t$ is

$$\varepsilon_0 = t^{-1} \int_0^t dx \int_0^1 w(\mu)\alpha(\mu, x) d\mu = \left\langle \alpha w \right\rangle_{x=0},$$

where $x$ is the coordinate where the fragment was produced, $w(\mu)$ is the angular distribution density of the fragments, $\mu = \cos\theta$, $\theta$ is the angle at which a fragment leaves the layer relative to the normal to the layer, and $\alpha(\mu, x)$ is the transmission coefficient. The following approximation is often used:

$$\alpha(\mu, x) = \begin{cases} 1 & \text{for } \mu > \mu_0; \\ 0 & \text{for } \mu < \mu_0; \end{cases}$$

$$\mu_0 = \xi [1 - (x/t)],$$
where $\xi = t/R$ is the reduced thickness of the layer in units of the fragment range $R$. This parameter is the main parameter in all ion stopping models [1, 4]. For an isotropic distribution of fission fragments $w(\mu) = 1$, and in approximation (3) we have [2]

$$\varepsilon_0 = 1 - (\xi/2).$$

In the present work, the effects due to multiple scattering arising when fragments accompanying fissioning of $^{235}$U nuclei by thermal neutrons pass through matter are estimated. Layers of uranium metal and oxide, 50–400 $\mu$/cm$^2$, thick, are studied. The estimates are obtained using the SRIM computer code (version 2003.26) to simulate the stopping of ions in a material [5]. The two-fragment approximation is used: a light fragment $^{97}$Mo and a heavy fragment $^{139}$Ba.

When using the SRIM program, there arises the question of whether or not the simulation of fission products in transport problems by the corresponding ions is correct. The data on the range of the fission fragments from $^{235}$U in light media agree with the calculations using the SRIM program to within 10% [6]. The results of comparing the ranges of the fission fragments $^{99}$Mo and $^{140}$Ba [7, 8] in heavy media (Zr, Au, and U) with calculations likewise confirm the conclusion that the stopping of fission fragments in matter is similar to that of ions.

The energy dependence of the range $R(E)$ of fission fragments can be described by the Bohr–Wheeler approximation [1, 7]

$$R(E) = \bar{R} \left( \sqrt{E} - \sqrt{E_c} \right),$$

where $\bar{R}$ is a constant that depends on the mass and charge of a fragment and the atoms in the material of the stopping medium; $E_c$ is the threshold energy of a fragment below which stopping does not occur. The approximation $E_c = 0$ is often used. The SRIM program uses a more complicated dependence $R(E)$, which, however, can be described by formula (4) if the dependence of the threshold energy $E_c$ on the fragment energy is introduced. In the range $E > 10$ MeV, the range $R(E)$ for $^{139}$Ba ions in uranium and argon oxide has the form $R(E) \sim E^{0.59}$. Large discrepancies between the descriptions are observed only in the range $E < 10$ MeV, where nonionization losses, taken into account in the SRIM program, are important.

To take account of multiple scattering effects, it is necessary to introduce into the model the coefficients of reflection $\beta(\mu, x)$ of the fission products from the substrate. The probability $\varepsilon_0$ of emanation from the layer (in the two-fragment approximation) of at least one of the two fragments is

$$\varepsilon_0 = \left(1/2\right) \sum_{k=1}^{2} \langle\alpha_k\rangle_{\mu x} + \langle\beta_k\rangle_{\mu x} - \langle\alpha_k\beta_{k-1}\rangle_{\mu x},$$

where the brackets $\langle \rangle$ denote averaging over the variables $x$ and $\mu$. We note that in general the following inequality holds for the probability of events in which a fragment $i$ flies out at the same time as the fragment $j$ is backscattered:

$$\langle\alpha_i\beta_j\rangle_{\mu x} \neq \langle\alpha_i\rangle_{\mu x} \langle\beta_j\rangle_{\mu x}.$$ 

Consequently, to estimate $\varepsilon_0$ it is necessary to know the angular dependences of the coefficients $\alpha(\mu, x)$ and $\beta(\mu, x)$.

It is known that multiple scattering is most strongly manifested for glancing angles of incidence of ions on a layer of matter. The SRIM program permits investigating in detail its angular dependence and the yield of backscattering for layers with different composition and geometry. Figure 1 shows the computational results obtained with the SRIM program for the transmission coefficients $\alpha(\mu)$ and reflection coefficients $\beta(\mu)$ for $^{139}$Ba ions in a layer of uranium oxide 63 and 125 $\mu$/cm$^2$ thick. For normal incidence on the layer, the contribution of backscattering does not exceed 0.01%, and it increases rapidly in the region $\mu < 0.1$. For a 125 $\mu$/cm$^2$ thick layer of uranium oxide, 99% of all backscattering events lie in the range $\mu < 0.2$ (88% for $\mu < 0.08$). A similar situation is also observed for $^{97}$Mo ions.

In the absence of multiple scattering, the transmission curve has the form

$$\alpha(\mu) = 0.5 \left[ 1 + \Phi((\mu - \xi)/(\eta\mu)) \right].$$

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