ANALYSIS OF THE DANGER OF STRUCTURAL FAILURE WITH UNCERTAINITIES PRESENT IN THE INITIAL DATA

V. V. Tkachev and K. K. Zheltukhin

Initial data on the properties of a material, which are often difficult to determine with prescribed accuracy because of large variances in the experimental results, for example, experimental determination of the characteristics of creep in long-time strength of the materials used in the core of a nuclear reactor, are needed for computational simulation of structural deformation and failure. These conditions engender uncertainty in the results of strength calculations, making them improperly posed from the mathematical standpoint. A mathematical method for taking account of the uncertainties in strength calculations is examined. The method is based on the reconstruction of the distributions of random quantities over a sample of empirical data, using nonparametric method of ordered minimization of risk. The method and the computer codes DENSIT, PTUBL, and PCLAD, which are used for performing the calculation, are described. The application of these codes to the analysis of an accident which occurred in the No. 3 unit of the Leningrad nuclear power plant in 1992 is examined.

In strength calculations, it is ordinarily assumed that a structure fails by two alternative mechanisms: as a result of the development of macroscopic defects (cracks) or the accumulation of submicroscopic damage during deformation. Conventionally, one talks about calculations of the brittle strength in the first case and the exhaustion of deformability in the second case.

In any case, initial data on the properties of the structural material, which are often difficult to determine with a prescribed accuracy because of the large variances in the experimental results, are needed for accurate calculations. This situation occurs, for example, in the experimental determination of the characteristics of creep and long-time strength of the materials used in the core of a nuclear reactor. The reason is that impurities, the fabrication technology, measurement errors, and instability of the process of deformation at high temperature and in the presence of irradiation influence the properties of materials and the experimental results. These conditions can engender uncertainty in strength calculations, making them improperly posed from the mathematical standpoint. Probabilistic methods are one of the recommended techniques for overcoming the improper nature of the formulation [1]. However, even with these methods difficulties arise with the initial data, which are prepared by statistical analysis of experimental results.

We shall examine the application of the most widely used approach to statistical analysis, based on the reconstruction of the distribution of random quantities by sampling the empirical data [2].

Ordinarily, parametric methods are used to reconstruct distributions. A hypothesis concerning the form of the distribution is introduced. For example, it is assumed that the distribution is normal or some other known distribution determined by a finite number of parameters. Then, these parameters are calculated by an appropriate analysis of the sample data using the method recommended for the distributions adopted [3]. The accuracy of the reconstruction of the distribution depends strongly not only on the sample elements but also on the hypothesis adopted about the distribution law. As a rule, there are not enough data to adopt such a hypothesis.
In the present article, we describe a mathematical approach based on a nonparametric method – the method of ordered minimization of risk, making it possible to overcome the difficulties arising. A simulation of the damage to fuel elements and tubes of fuel channels in a RBMK reactor under accident conditions is examined as an example.

The foundations of the theory of the reconstruction of distributions by a nonparametric method were laid in [4, 5]. They are presented in their final mathematically substantiated form in [6, 7].

We shall reconstruct the distribution of random quantities in a family of linear quantities using a vector parameter $\alpha$ of the functions

$$F(x, \alpha) = \sum_{j=1}^{N} \alpha_j f_j(x),$$

where $\alpha_j$ are the components of the vector $\alpha$ which are to be determined, and $f_j(x)$ and $N$ are the trial functions and their number, respectively.

The components of the vector parameter $\alpha$ can be found by minimizing the empirical average-risk functional [6]

$$I(\alpha) = \int [y - F(x, \alpha)]^2 dP(x, y),$$

where $y = F_n(x)$ is the desired empirical distribution function of the random parameter $x$ that is constructed according to an experimental sample containing $n$ observations of the realization of a random variable $x$ (we note that in the general case the parameter $x$ can be multidimensional); $P(x, y)$ is the joint probability density of the random variables $x, y$ which characterizes the measure in the $x$-$y$ space (we note that in contrast to the desired distribution function $y = F(x)$ the empirical function $y = F_n(x)$ is a random quantity which depends on the number of observations $n$).

In the numerical solution, the empirical average-risk functional

$$I_p(\alpha) = \frac{1}{n} \sum_{i=1}^{n} [y_i - F(x_i, \alpha)^2 / K$$

is minimized instead of minimizing the (2) functional [7]. Here $x_i$ and $y_i$ are elements of a random and independent sample of size $n$, which consist of values of the random parameter $x$ and the values of the empirical distribution function $y_i = F_n(x_i, \alpha)$ associated with them; $K$ is a coefficient that depends on the form of the family of functions $F(x, \alpha)$.

Euler’s method for minimizing the (3) functional is well-known and reduces to solving a system of linear algebraic equations for the components of the vector $\alpha$.

The following estimates of the accuracy of the reconstruction of distributions by the method of ordered minimization of risk are presented in [7].

The error $\varepsilon$ in reconstructing the distribution on the basis of the mean-square deviation $I_p^2$ is estimated by the following inequality:

$$\sqrt{\int [y - F(x, \alpha_{opt})]^2 dP(x, y)} \leq \sqrt{I_p(\alpha_{opt})} = \varepsilon,$$

i.e., by the empirical-risk functional which is calculated after minimization has been performed and the optimal vector $\alpha_{opt}$ has been determined with optimal number of trial functions $N$ which minimize the functional.

The most conservative method for estimating the error of the reconstruction of a distribution is an estimate based on the maximum deviation (excursion). According to Kolmogorov’s criterion,

$$\varepsilon_n = \|F_n(x) - F(x)\|;$$

$$\varepsilon = \sup \|F_n(x) - F(x)\|;$$