A general principle is proposed for introducing multicomponent potentials by summing the separative power of the elements that divide the partial flows of binary mixtures consisting of pairs or sets of isotopes. It is shown that under different assumptions about the character of the separation process this method can be used to determine the well-known as well as new forms of separation potentials. A computational experiment is used to determine the potentials that best correspond to the separative processes in R- cascades and optimal cascades with minimum total flow and prescribed external concentration of the target isotope.

At present, there is no standard interpretation of the separation potential. There are many approaches and works devoted to determining it. The seminal works were by P. Dirac and K. Cohen, who introduced the potential for a binary mixture of isotopes on the basis of an axiomatic approach and analysis of an ideal cascade [1]. Conflicts between the initial assumptions and the actual results arise when attempts are made to extend the axiomatic approach to multicomponent separation [2]. Many investigators have proposed a new interpretation of the axiom in order to overcome these difficulties. As a result many, appreciably different, potentials based on different assumptions have been introduced.

In the present work, a general principle is proposed for introducing multicomponent potentials by summing the separative powers of the elements separating the partial flows of binary mixtures comprised of pairs or sets of isotopes. The best known potentials and new forms of these potentials are determined with different assumptions about the character of the separation performed by this method. A computational experiment was used to determine potentials that best correspond to the separation processes in R-cascades and cascades optimized for minimum total flow with the prescribed external concentration of the target isotope.

**Particulars of Separation Potential Determination.** The value of an isotopic mixture is determined by a function $U$ that can be represented as the product of the amount of a mixture and a separation potential $V(C_1, C_2, ..., C_m)$ that depends on the isotopic concentrations $C_i (i = 1, ..., m)$ and characterizes the quality of a mixture. The quantity $m$ is the number of components in the mixture, which we shall enumerate in order of increase molecular mass. Accordingly, a change in the value function in the separative apparatus (element, step or cascade) is collated with the useful separative work. If the rate of change of the value function is the focal point, then its value determines the separative power of the apparatus.

The separative power for steps $\Delta U_s$ and cascades $\Delta U_c$ is calculated similarly.

\[
\delta U = L'V(C'_1, C'_2, ..., C'_m) + L''V(C''_1, C''_2, ..., C''_m) - LV(C_1, C_2, ..., C_m),
\]

where $L'$, $L''$, and $L$ are the product, waste, and feed flows, respectively; and $C'_i$, $C''_i$, and $C_i$, $i = 1, m$, are the corresponding isotopic concentrations. The fraction enriched with the light isotope is the product, and the depleted fraction is the waste.
Since the component sum of the isotopic concentrations equals 1, the following notation is introduced: $C_1 = C$ and $C_2 = 1 - C$. The process is characterized by the total separation factor defined as $q = C'(1 - C'')/[(1 - C')C'']$. For low enrichment ($q \sim 1$), Dirac’s axioms are used to find the separation potential. The first axiom states that the potential is a function of the concentration $C$ only; the second one states that the separative power of an element is independent of the concentration. Solving the functional equation $\delta U = \text{constant}$ we obtain \[ V(C) = (2C - 1)\ln \frac{C}{1-C}. \] (2)

A similar expression for the potential is obtained for arbitrary symmetric enrichment, where the separation factors $\alpha$ and $\beta$ based on the product and waste, respectively, are equal to one another: $\alpha = C'(1 - C)/[(1 - C')C]$ and $\beta = C(1 - C'')/[(1 - C)C'']$. It is impossible to satisfy both axioms in the asymmetric case $\alpha \neq \beta$. For this reason, the concept of the separation power $\delta U_1$ and $\delta U_2$ calculated for the partial flow of the components is introduced \[ \delta U_k = L'_k V_k(C_k') + L''_k V_k(C_k'') - L_k V_k(C_k) = L_k f_k(\alpha, \beta), \quad k = 1, 2, \] (3)

where $f_1(\alpha, \beta) = f_2(\beta, \alpha) = (\alpha/\beta - 1) \ln(\alpha) - (\alpha - 1)\ln(\beta)/(q - 1)$; $L'_k$, $L''_k$, and $L_k$ are the flows; and $V_k$ is the separation potential of the $k$th component. These quantities are given by

\begin{align*}
L'_k &= LC_k'; \\
L''_k &= L'C_k''; \\
L_k &= LC_k; \\
V_1(C_1) &= \ln[C/(1 - C)]; \\
V_2(C_2) &= \ln[(1 - C)/C].
\end{align*}

To achieve independence from concentration, $\delta U$ was represented as a linear combination \[ \delta U = f(\alpha, \beta)[\delta U_1/f_1(\alpha, \beta) + \delta U_2/f_2(\alpha, \beta)] = f(\alpha, \beta)(L_1 + L_2) = f(\alpha, \beta)L, \] (4)

where $f(\alpha, \beta)$ is an arbitrary function, which it is convenient to take as $f_1(\alpha, \beta)$ or $f_2(\alpha, \beta)$. Hence the separation potential was determined as

\[ V(C) = f(\alpha, \beta)[C/f_1(\alpha, \beta) - (1 - C)/f_2(\alpha, \beta)]\ln[C/(1 - C)], \]

which is identical to the function in \[4\] and can be used to optimize a cascade so as to maximize the separation power of the elements \[5\].

The principle of summing the separation power can be extended to multicomponent mixtures. In its most general form, the separative power of an element and other devices can be represented as

\[ \delta U = \sum_{i,j} \delta U_{ij} p_{ij}, \] (5)

where the summation indices $i$ and $j$ determine the isotopic pair or set chosen; $\delta U_{ij}$ is the separation power for the partial flows of the $i$, $j$th pair; and $p_{ij}$ is a coefficient depending on the character of the separation.

**Separation Potentials Depending Only on the Isotopic Concentration.** Multicomponent separation can be represented as a sum of $m(m - 1)/2$ independent, parallel, separation cycles, in each of which a pair of isotopes is separated. Such a separation into cycles is impossible in practice; this representation is a mathematical device.

We shall determine the separation potential on the basis of general relation (5) with identical coefficients $p_{ij} = 1$ for the separation power corresponding to a cycle. We shall assume that only a mixture of the isotopic pairs of interest participates in each separation cycle. For a pair of isotopes with the indices $i$ and $j$ (let $i < j$), the feed flow of the element separating this mixture is $L_{ij} = L(C_i + C_j)$. The product $L'_{ij}$ and waste $L''_{ij}$ flows are determined similarly. We now introduce