Mathematical Description of Human Body Constitution and Fatness

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Using mathematical modeling of human body, we demonstrated logical drawbacks of body mass index \( BMI_1 = \frac{M}{H^2} \) (A. Quetelet, 1832) and proposed more precise body mass index \( BMI_2 = \frac{M}{H^3} \) as well as body constitution index \( BCI = \left( \frac{M}{H^3} \right)^{1/2} \) and fatness index \( FI = \frac{M}{H^2C^2} \), where \( M, H, \) and \( C \) are body weight, height, and wrist circumference of the individual.

Key Words: body mass index; body constitution index; fatness index

According to WHO standards [3-5], human nutritional status is mainly judged from empirical body mass index \( BMI_1 = \frac{M}{H^2} \) (1), where \( M \) and \( H \) are body weight and height (A. Quetelet, 1832) [2]. This means that the relative body fat is determined by the amount of non-uniform body weight per arbitrary area unit \( H^2 \) that strongly depends on the body constitution and is not consistent with actual body surface area \( S = d(\text{MH})^{1/2} \) (2), where \( d \) is a gender quotient [1]. Therefore, the aim of our study was to search for a new parameter adequately reflecting body fatness for any individual height and weight.

MATERIALS AND METHODS

Methodological premise was the assumption that the body surface area is described by the equation \( S = cV^{2/3} = c(M^2/\rho^2/3) \) (3), where \( V, \rho, \) and \( c \) are the volume, density, the shape coefficient, respectively. With regard to humans, this formula agrees with the concept of M. V. Chernorutskii (1928), according to which all people are morphologically divided into asthenics, normosthenics, and hypersthenics depending on the ratio of longitudinal and transverse dimensions of the skeleton. Therefore, in terms of spatial modeling, each asthenic or hypersthenic is “stretched” or “flattened” in height normosthenic of the same volume and fatness. At the same time, each body type can include normal, over-, and under-nourished subjects. To identify them, the corresponding indices were found.

To test the efficiency of these indices, young women \((N=154)\) and young men \((N=58)\) aged from 17 to 22 years and selected considering the rules \( 3\sigma \) were surveyed. Height \((H, \text{cm})\), weight \((M, \text{kg})\), and wrist circumference \((C, \text{cm})\) were measured in all subjects; the arithmetic mean \((\bar{M})\), standard deviation \((\sigma)\), and standard error \((m)\) of these parameters and those obtained in the course of the study were calculated. The interval from 0.9 to 1.1 \( \bar{M} \) was taken as the age and gender norms in both samples, because it exactly matched the range \((\bar{M} \pm 3\sigma)\) for \( M, H, C \), and of the surveyed individuals.

RESULTS

If we substitute the value \( S \) of equation (2) into equation (3), we shall find that \( c = d(H^{1/2}M^{1/2})(M^{2/3}/\rho^{2/3}) = d\rho^{2/3}(M/H^3)^{1/6} \) (4) or \( d\rho^{4/3}c^2 = M/H^3 \) (5). From this, the morphometric description of humans includes the ratio of body weight to height, but it is raised not to the second power, as in equation (1), but to the third power. In this case, the complex expression \((d\rho^4/c^2)\) is a new body mass index \( BMI_2 = M/H^3 \) (6) reflecting body weight excess or deficiency according to gender (coefficient \( d \)), body shape (coefficient \( c \)) and body fatness (coefficient \( \rho \)). Considering that only body weight is the function in equation (6), the more correct
variant should be \( M = (BM_1)H^3 \) reflecting expected dependence of body weight on height in a normosthenic value of \( BM_1 \). Discrepancy of the actual and expected body weight can be determined by nonstandard habitus and/or abnormal fatness.

In this case, for evaluation of specific human body type, the body weight should be divided by its height and density which will give an average cross-sectional area \( A \) of the body, square root of which is the average cross-sectional area \( D/H \). This means that the ratio \( D/H \) is the index of body shape (normal, “flattened” or “stretched”) that completely coincides with the above body types. In this case, body constitution index (BCI) looks like \( M/HC^2 \) or simply like \( BCI = (BM_1)H^3/\rho^{1/2} \) (7), if the unimportant (by magnitude) coefficient \( \rho^{1/2} \) should be removed.

At the same time, this assessment of body constitution well suits only for normally-fed people, whereas asthenic with high body fat or hypersthenic with low body fat could be erroneously attributed to normosthenics due to \( BM_1 \) and BCI shifted to the normal values for these indices. Therefore, a simple way of independent evaluation of the body fatness was developed to clarify the constitution of these pseudonormosthenics.

According to this approach, at uniform “flattening” or “stretching” of the normosthenic, all areas of any section of the human body and in particular cross-sectional area in the wrist \( B \) simultaneously vary in the same way along with the dynamics of the average area \( A \). Hence, the \( A/B \) ratio remains constant for any section of the human body and in particular cross-sectional area \( A \) of the body, square root of which is the average cross-sectional area \( D/H \). This means that the ratio \( A/B \) is the index of body fatness (FI) irrespective of the body constitution.

To calculate this indicator, it is necessary to take cross-sectional area of the wrist as a circle and then the index can be expressed by the equation: \( FI = A/B = (M/\rho H)/(\pi R^2) \) (8), where \( R \), the radius of wrist circumference \( C \). Continuing the transformation, we get \( FI = (4\pi/p)(M/HC^2) \) (9). Finally, removal of inessential coefficients \( 4\pi \) and \( p \) yields final \( FI = M/HC^2 \) (10), where \( H \) and \( C \) are for convenience expressed in dm.

The resulting indicator immediately clarifies the type of the body constitution in pseudo-normosthenics, because increased FI under normal \( BM_1 \) and BCI is possible only in asthenics with high body fat, and reduced FI is possible in hypersthenics with low body fat.

The results of gender-based indexes are presented in Table 1. In this case, normal (i.e., not exceeding 0.9-1.1 \( M_{154} \)) \( BM_1 \) was found only in 92 young women therefore allocated to group A. At the same time, formal body weight deficiency or excess was observed in 32 and 30 persons, allocated to groups B and C, respectively.

According to BCI, these women were classified as follows: obvious asthenics (BCI below normal), 9 obvious hypersthenics (BCI above normal), and 138 true normosthenics and pseudo-normosthenics (BCI within the normal range).

According to FI, we revealed 114 young women with normal body fat, 20 women with high body fat (FI above normal), and 20 with low body fat (FI below normal).

Analysis showed that group B (BMI1 below normal; Table 2) included obvious asthenics (among them, 3 individuals with FI below normal), all the normosthenics with high body fat (4 persons), and 21 persons with normal BCI and FI with the formal body weight deficiency. Thus, the mean values of BCI \( (M_{21} \pm m = 3.324 \pm 0.011 \) arb. units) and FI \( (M_{21} \pm m = 1.431 \pm 0.018 \) arb. units in this subgroup were shifted to the lower boundary of the normal range between \( M_{154} \) and \( 0.9M_{154} \), which determined reduced \( BM_1 \). Therefore, all these young women were attributed to the medium-variant of body constitution (semiasthenics).

Conclusive evidence of morphological proximity of obvious asthenics and semiasthenics was the virtually identical wrist circumference amounted to 14.43±0.17 (\( M_{21} \pm m \)) and 14.60±0.14 cm (\( M_{154} \pm m \)), respectively.

The similar analysis of group C (BMI1 more than normal, Table 2) showed that it comprised all the obvious hypersthenics (including 5 persons with

<table>
<thead>
<tr>
<th>Group</th>
<th>Young women, ( N=154 )</th>
<th>Young men, ( N=58 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age, years</td>
<td>18.8±0.1</td>
<td>18.8±0.1</td>
</tr>
<tr>
<td>Height, cm</td>
<td>164.3±0.5</td>
<td>177.1±1.0</td>
</tr>
<tr>
<td>Weight, kg</td>
<td>56.55±0.61</td>
<td>69.27±1.50</td>
</tr>
<tr>
<td>WC, cm</td>
<td>15.3±0.1</td>
<td>17.2±0.1</td>
</tr>
<tr>
<td>( BM_1 ), kg/m²</td>
<td>20.909±0.188</td>
<td>22.056±0.421</td>
</tr>
<tr>
<td>( BM_1 ), kg/m³</td>
<td>12.739±0.120</td>
<td>14.797±0.251</td>
</tr>
<tr>
<td>BCI, arb. units</td>
<td>3.563±0.017</td>
<td>3.523±0.035</td>
</tr>
<tr>
<td>FI, arb. units</td>
<td>1.473±0.011</td>
<td>1.323±0.019</td>
</tr>
</tbody>
</table>

Note. WC, wrist circumference. When determination FI, height and wrist circumference were measured in dm. \( p=0.016, **p=0.001 \).