ESTIMATION OF THE LAGRANGIAN VELOCITY STRUCTURE FUNCTION CONSTANT $C_0$ BY LARGE-EDDY SIMULATION

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Abstract. The inertial subrange Kolmogorov constant $C_0$, which determines the effective turbulent diffusion in velocity space, plays an important role in the Lagrangian modelling of pollutants. A wide range of values of the constant are found in the literature, most of them determined at low Reynolds number and/or under different assumptions. Here we estimate the constant $C_0$ by tracking an ensemble of Lagrangian particles in a planetary boundary layer simulated with a large-eddy simulation model and analysing the Lagrangian velocity structure function in the inertial subrange. The advantage of this technique is that it easily allows Reynolds numbers to be achieved typical of convective turbulent flows. Our estimates of $C_0$ is $C_0 = 4.3 \pm 0.3$ consistent with values found in the literature.

Keywords: Lagrangian dispersion, Lagrangian stochastic models, Lagrangian velocity structure function, Large-eddy simulations, Planetary boundary layer.

1. Introduction

Since the pioneering work of Taylor (1921) Lagrangian modelling of fluid motion is of fundamental importance in studies of mixing and dispersion. In this context the constant $C_0$ determines diffusion in the velocity space and in particular plays an important role.

According to the Kolmogorov scaling at high Reynolds numbers in the inertial subrange the second-order Lagrangian velocity structure function assumes the form:

$$D_2^L(\tau) = \langle [u(t + \tau) - u(t)]^2 \rangle \equiv C_0 \bar{\varepsilon} \tau$$

(1)

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for \( \tau_\eta \leq \tau \leq T_L \) where \( u(t) \) is the Lagrangian velocity, \( \langle \rangle \) denotes the ensemble average, \( \tau \) is a time lag, \( \bar{\varepsilon} \) is the average dissipation rate of kinetic energy, \( C_0 \) is a constant, \( \tau_\eta \) is the Kolmogorov time scale (at which dissipation starts to be effective) and \( T_L \) is the Lagrangian integral time scale.

The importance of the constant \( C_0 \) stems from the fact that predictions of turbulent dispersion by means of Lagrangian stochastic models (LSM) depend upon its value (Thomson, 1987; Sawford and Guest, 1988; Luhar and Britter, 1989; Weil, 1990; Du et al., 1994; Rotach et al., 1996; Wilson and Sawford, 1996; Ferrero and Anfossi, 1998; Degrazia and Anfossi, 1998; Anfossi et al., 2000). In these models it is assumed that the evolution of a tracer particle’s state (velocity-position) is a Markovian process, so the particle’s trajectory can be statistically calculated from (Thomson, 1987):

\[
du_i = a_i(x, u, t)dt + b_{ij}(x, u, t)d\xi_j,
\]

where \( a_i(x, u, t) \) is the particle’s acceleration in direction \( i \) and \( b_{ij}(x, u, t)d\xi_j \) is a random forcing caused by the fluctuating pressure gradients and molecular diffusion. To be consistent with Equation (1) the term \( b_{ij} \) should be equal to (Monin and Yaglom, 1975; Pope, 1987):

\[
b_{ij} = \sqrt{C_0\bar{\varepsilon}\delta_{ij}},
\]

where \( \delta_{ij} \) is the Kronecker delta. This implies that the evaluation of the constant \( C_0 \) is a fundamental issue in Lagrangian stochastic modelling.

The numerical value of \( C_0 \) has been determined in different ways, including from Lagrangian velocity measurements, from numerical experiments, from observed dispersion of tracer particles in a flow, from Eulerian measurements, and based on theoretical assumptions. This explains the wide range of values found and the fact that its value is rather uncertain and still a matter of discussion (Anfossi et al., 2000; Yeung, 2002; Lien and D'Asaro, 2002).

Using Lagrangian measurements the \( C_0 \) value can be estimated from the level of the plateau of the \( D_L^2(\tau)/\bar{\varepsilon}\tau \) versus \( \tau \) curve. It has been shown (Mordant et al., 2001; Yeung, 2002) that for insufficiently high Reynolds numbers the plateau may be very short or it may exist only as “bumps”. Mordant et al. (2001) measured the Lagrangian velocity of tracer particles in the laboratory in a range of microscale Reynolds number \( R_\lambda \) between 100 and 1100. The structure functions \( D_L^2(\tau)/\bar{\varepsilon}\tau \) obtained do not show a plateau but they reach a maximum with a level in the range between 0.5 and 4.

From experimental analyses, observations of Lagrangian (neutrally buoyant balloons) trajectories in the atmosphere made by Hanna (1981) and analysed by Thomson (1987) gave \( C_0 = 4 \pm 2 \).

A series of direct numerical simulations (DNS) with increased \( R_\lambda \) have yielded a wealth of information on Lagrangian statistics (Sawford and Yeung, 2001; Yeung, 2001). The most recent attempts to extrapolate the...