The maximum apparent level of liquid in a vessel during a transient process at an assigned rate of pressure reduction is calculated. Curves of the magnitude of “swelling” of liquid methane stored in 6- and 25-m³ vessels versus the rate of reduction in their initial 0.2- and 0.6-MPa pressures are presented. The maximum allowable rate of pressure reduction in the vessel can be determined in this manner.

A pressure reduction in a vessel filled with a cryogenic liquid is frequently accompanied by the ejection of a gas–liquid mixture through a drainage pipeline with a short-lived pressure increase in the vessel. The cause of this phenomenon is vaporization in the volume of the liquid during the pressure drop (boiling process), which is accompanied by an increase in the volume of the liquid (“swelling” effect).

The “swelling” effect during vaporization or bubbling in a stationary regime is rather well understood. A characteristic feature of the boiling process as the pressure drops in a vessel is the lack of an external heat load and a shift in the liquid–vapor boundary (surface of the liquid) in the initial stage; it is therefore necessary to analyze the process in the non-stationary statement under adiabatic conditions.

The problem of the boiling of a cryogenic liquid in a vessel can be formulated in the following manner. In a partially filled adiabatic vessel, the initial pressure is maintained. At time \( \tau = 0 \), the pressure begins to drop, whereupon vaporization takes place in the volume of the liquid, and the apparent level of liquid begins to rise.

The displacement rate of the liquid–vapor boundary is governed by the bulk rate of vapor generation on the one hand, and by the bulk flow rate of vapor through the surface of the liquid on the other. When these parameters become equal, the transient process is terminated, and a quasi-stationary regime is established.

In our study, we determine by computational means the maximum apparent liquid level in a vessel during the transient process at a given rate of reduction in pressure. The problem was solved for the following assumptions [1]: the liquid and vapor are in thermodynamic equilibrium; the thermophysical properties of the liquid and vapor are constants, and are described by the Clapeyron–Clausius equation; the variation in the mass of liquid in the vessel during the transient process is negligibly small; and the cross-sectional area of the vessel is constant, as is the rate at which the pressure drops. As a working hypothesis, it is assumed that the travel speed of the vapor relative to the liquid is equal to the group velocity at which the vapor bubbles are rising to the surface.

When vapor is generated in the volume and the bubbles rise to the surface, the cross-sectional area of the vessel occupied by the vapor gradually increases, i.e., the actual vapor content, the value of which can be determined from the equa-
tion of mass conservation for the vapor phase, which assumes the following form for section \( z \) and a relative vapor travel speed \( w_V \), increases in each section:

\[
\frac{\partial (p_V f \phi)}{\partial \tau} + w_V \frac{\partial (p_V f \phi)}{\partial z} = m;
\]

where \( p \) is the pressure in the vessel, \( T_S \) is the equilibrium temperature of the liquid, \( \tau \) is time, \( r \) is the heat of vaporization, \( c_s \) is the heat capacity of the liquid, \( \rho_L \) is the density of the liquid, \( \rho_V \) is the density of the vapor, \( f \) is the cross-sectional area of the vessel, \( \phi \) is the actual vapor content, \( f \phi \) is the cross-sectional area of the vessel in section \( z \) occupied by vapor, \( f(1 - \phi) \) is the cross-sectional area of the vessel in section \( z \) occupied by liquid, \( w_V \) is the group travel speed of the rising vapor bubbles, and \( m \) is the internal source of mass in section \( z \) (the value of \( m \) is the rate of vapor generation in section \( z \) as the pressure drops).

The group velocity at which the bubbles rise to the surface is determined from the formula [2]

\[
w_V = \psi w_p,
\]

where

\[
w_p = 1.5 \sqrt{\frac{g \sigma}{\rho \left( \frac{\rho_L - \rho_V}{\rho_L^2} \right)}}
\]

is the rate of ascent of a single bubble;

\[
\psi = 1.4 \left( \frac{\rho_L}{\rho_V} \right)^{0.2} \left( 1 - \frac{\rho_V}{\rho_L} \right)^5
\]

is the bubble-interaction factor; \( \sigma \) is the surface tension of the liquid; and \( g \) is the acceleration of free fall.

For a constant rate of bubble ascent, cross-sectional area and vapor density (the vapor density is independent of the hydrostatic pressure), it is possible with the use of the Clapeyron–Clausius equation to reduce Eqs. (1) and (2) to the form

\[
\frac{\partial \phi}{\partial \tau} + w_V \frac{\partial \phi}{\partial z} = (1 - \phi)K_p p_t,
\]

where

\[
K_p = \frac{c_s T_s \rho_L}{r^2 \rho_V^2},
\]

\( p_t \) is the rate at which the pressure drops.

Equation (4) describes the variation in the actual vapor content in each section over the height of the liquid and during the time that vapor is being generated due to a pressure drop. This equation is quasi-linear, and the method of characteristics can be used for its solution. As a result, we obtain

\[
\phi = 1 - \exp(-K_p p_t \tau) \quad 0 \leq \tau \leq \frac{z}{w_V};
\]

\[
\phi = 1 - \exp(-K_p p_t \frac{z}{w_V}) \quad \tau \geq \frac{z}{w_V}.
\]