IMPROVING THE ACCURACY OF HYDROCYCLONE ANALYSIS BY ACCOUNTING FOR TRANSFER OF HYDRODYNAMIC LOSSES FROM THE BOUNDARY LAYER TO THE CORE OF THE FLOW

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It is noted that losses during flow along the surface is the basic form of loss in hydrocyclones. The special role of the axisymmetric frontal effect, which results in vigorous longitudinal circulation that transfers losses from the boundary layer to the core of the flow, is demonstrated. The velocity and pressure distribution along the radius is determined from the equality of the losses on the surface and in the core of the flow, but in the axial direction, it is determined on the basis of the principle of maximum flow rate. A model of the flow in the conical zone of a hydrocyclone, which makes it possible to determine the narrowing of the air column toward the vertex of the cone and the flow rate through the slime opening is created. An explanation is given for a number of anomalous hydrodynamic phenomena.

When the most accurate procedures are used to analyze hydrocyclones, the mean-square error generated in calculating the overall output is currently more than 20% [1], and the computed and experimental outputs with respect to slimes frequently differ by several factors one from the other. One of the causes of this noncorrespondence is the absence of an accounting of the friction induced as the liquid flow moves over the developed surface of the hydrocyclone (HC). Fominykh and Mochalkin [2], and Zhityannyi and Khusainov [3] have experimentally established, therefore, that an increase in the size and roughness of the surface being washed will lead to an increase in the flow rate of the feed; this is uncharacteristic of the majority of hydraulic equipment, and is anomalous.

Other paradoxical functional relationships, for example, an increase in the flow rate of the suspension with its increasing density and viscosity (to certain limits) may also qualify. For certain combinations of basic geometric HC dimensions, a reduction in the size of the slime opening will lead to a reduction in the flow rate of suspension at first, and then again to its sharp increase [4].

For the high Reynolds numbers characteristic of HC with a water-dispersion medium, the predominant part of hydraulic losses is, according to classical canons, concentrated in the thin boundary layer beyond the bounds of which the liquid can be treated as ideal with a certain degree of approximation, and for which the “free-vortex” law

\[ v_\theta r = \text{const} \]  

is observed, where \( v_\theta \) is the tangential component of the circumferential velocity of the liquid (suspension), and \( r \) is the radius.

It is well known, however, that the circumferential velocities in HC deviate widely from law (1). This deviation and significant hydrodynamic losses in the core of the flow with small losses at the inlet to the HC, and the absence of classical conditions for separation of the boundary layer in the HC chamber enable us to suggest that the transfer mechanism of losses from the boundary to the core of the flows is non-classical.
It is convenient to express the losses on the surface of the HC in terms of tangential stresses. When the flow moves in a constant-section channel, the tangential stress on the wall is determined from the formula \( \tau = \frac{1}{8} \frac{\lambda \rho v^2}{F} \), where \( \rho \) and \( v \) are the density and velocity of the liquid, and \( \lambda \) is the coefficient of hydraulic friction in the Darcy–Weisbach formula. This formula is used in the theory of centrifugal atomizers to account for the effect of viscous friction against the walls on the velocity profile and flow characteristics of the atomizers. Despite the fact that the velocity is constant within the channels of atomizers, use of this formula will make it possible to obtain good convergence between computed and experimental results; this is the basis for use of a similar approach to HC.

In the core of the flow, let the liquid move in conformity with law (1); the axial and radial velocities can then be neglected. Let us designate the integral friction horsepower in the boundary layer, i.e., over the entire surface of the HC, in terms of the input hydraulic power, by the surface-friction factor \( \Psi \) (SFF). Using the above-cited expression for the stress \( \tau \):

\[
\Psi = \frac{\lambda \int F \rho v^3 \, dF}{\rho_1^* Q_1} = \frac{\frac{\lambda \pi}{2} \left( L + \frac{L_2}{d_2^*} + \frac{1}{1 + \frac{1}{\tan(\alpha/2)} \left( \frac{1}{d_3^*} - 1 \right)} \right) \rho_1^*}{1 - \frac{1}{\tan(\alpha/2)} \left( \frac{1}{d_3^*} - 1 \right) \rho_1^*},
\]

where \( \rho_1^* \) and \( Q_1 \) are the total pressure and flow rate of liquid at the inlet to the apparatus; \( L \) and \( L_2 \) are the relative lengths of the cylindrical section and overflow pipe; and \( d_2^* \) and \( d_3^* \) are the relative diameters of the overflow and slime openings. The relative dimensions are dimensions with respect to the diameter of the cylindrical portion. Subscripts 1, 2, 3, and 0 apply, respectively, to the feed, overflow, and slime openings, and the air column.

For familiar HC designs, the SFF fluctuate from 0.2–0.4 to 3–5. The larger values are obtained for small tapers of the cone and relative diameters of the feed and discharge openings, and the larger lengths to the cylindrical section and overflow pipe. For small SFF values, disregard of losses on the surface of the HC will not result in significant errors. When \( \Psi > 1 \), they must be accounted for, and in the opposite case, the power of the losses against the walls may exceed the hydraulic-power input, which is physically impossible.

Valyukhov et al. [5, 6] indicate the determining role of the asymmetric frontal effect. A near-wall centripetal flow develops on the frontal surface of the HC due to the zero velocity of the liquid, and uncompensated centrifugal forces in the core of the flow [7]. In the conical sections, any surface element between radii \( r \) and \( r + dr \) will be larger than a corresponding surface element of the front wall by a factor of \( \sin(\alpha/2) \); the energy scale of the surface phenomena will therefore be higher. Here, different radial velocity and pressure profiles will develop at opposite ends of the HC. The difference in the static pressures against the faces will result in the development of longitudinal flows that transfer losses from the boundary layer to the core of the flow; this will also cause the tangential component of the circumferential velocity of the flow to deviate from the law of “vortex conservation,” which is known from numerous experimental data.

Let us examine a mathematic model of the flow of liquid in an HC constructed on the assumption that secondary losses after separation of the flow from the frontal walls will be lower than those in the boundary layer, and are not considered; this model is expressed by the energy-balance equation

\[
p_1^* Q_1 - N_{fr} = p_2^* Q_2 + p_3^* Q_3,
\]

where \( N_{fr} \) are the total power losses on the surface of the HC.

In approximating the radial circumferential-velocity profile by a power function with an exponent \( n \), which can be called the circulation indicator, it is readily demonstrated that the losses of static \( \Delta p \) and total \( \Delta p^* \) pressures as the flow travels from the radius \( R \) of the wall to radius \( r \) (the difference between the corresponding pressures on the periphery and radius \( r \)) are determined by the formulas

\[
\Delta p = \frac{\rho v_R^2}{2n} \left[ \left( \frac{R}{r} \right)^{2n} - 1 \right]; \quad \Delta p^* = (1 - n)\Delta p,
\]

where \( v_R \) is the velocity of the liquid on the periphery of the flow.