EMPIRICAL ANALYSIS OF ESTIMATES OF REALIZED VOLATILITY IN FINANCIAL RISK CONTROL PROBLEMS

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Historic (realized) volatilities of the DAX and Dow Jones Industrial financial indices are calculated and analyzed using three different indicators. Recommendations for further use of the results are formulated.

Keywords: realized (historic) volatility, financial index, yield, high-frequency data, unevenly sample data.

INTRODUCTION

L. Bachelier, a French mathematician, was the first to try (in 1900) to describe a stock value as a random process. The Bachelier model \cite{1} has a number of erratic assumptions; however, it underlies the well-known formula P. Samuelson has derived to describe geometrical (also called economic) Brownian motion \cite{2}:

\begin{equation}
S_t = S_0 e^{\left(\frac{\mu - \sigma^2}{2}\right)t + \sigma W_t},
\end{equation}

where $S_t$ and $S_0$ are the share prices at times $t$ and 0, respectively, $\mu$ is the trend coefficient (average change of cost), and $\sigma$ is the volatility coefficient. Volatility is one of the fundamental concepts in the mathematical theory of finance. It is the basic risk measure for market financial instruments — shares, futures, options, etc.; it reflects their yield fluctuation. A high-level volatility means that yield varies within a wide range; a low-level volatility of assets suggests that its yield varies insignificantly. Assets with a lower level of volatility are less risky than assets with a higher level of volatility.

The formula of geometrical Brownian motion became the basis of the well-known Black–Scholes theory whose major result is the fundamental equation for the rational price $V = V(S, t)$ of a European-type option at a time $t$, with a strike price $K$ and exercise time $T$:

\begin{equation}
\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0,
\end{equation}

where $r$ is the interest rate of a riskfree asset, $0 \leq t \leq T$, and $S \geq 0$. The solution of Eq. (2) is given by

\begin{equation}
V(S, t) = S \Phi(d_1) - Ke^{-r(T-t)}(d_2),
\end{equation}

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where $\Phi(x)$ is the standard normal distribution function. The Black–Scholes theory [3] is based on several assumptions, of which the most important is that volatility $\sigma$ is a known constant. Actually, the unobservability of volatility is a huge problem of the modern finance. Therefore, application of the popular Black–Scholes theory requires that volatility be estimated.

There are two basic methods for estimation of the volatility of financial assets: (i) implied volatility, which uses the option prices actually declared in the market $\tilde{V}(S,t)$; being substituted into Eq. (3), the implied volatility can be found as a solution of the equation $\tilde{V}(S,t)=V(S,t,\sigma)$ and is a good estimate for the actual volatility; (ii) empirical, realized or historic volatility, which is based on statistical data — the past changes in the prices of a financial index. Capabilities for acquiring necessary statistical data are extremely wide nowadays; nevertheless, the problems of high-frequency and unevenly sampled data arise during data processing.

The purpose of this study is to empirically analyze and compare various approaches to the estimation of realized volatility for evenly sampled data, for two financial indices. Methods of evaluating realized volatility are described in Section 1. Section 2 contains the calculation results. Conclusions and recommendations as to the further application of the results are given in the final section.

1. METHODS OF EVALUATING REALIZED VOLATILITY

As indicated above, realized (historic) volatility reflects past variations in a share price or an index. There are several methods to estimate realized volatility.

The most simple approach is to use the concept of standard deviation: realized volatility can be calculated as the standard deviation of return on equity, calculated for a fixed time interval, i.e.,

$$\sigma_{st} = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (r_t - \bar{r})^2},$$

(4)

where $r_t = \ln \frac{C_t}{C_{t-1}}$ is the yield of the index calculated as the natural log of the ratio of close prices for the current and previous days, $C_t$ is the close price at a day $t$, $t=1:n$; and $\bar{r} = \frac{1}{n} \sum_{t=1}^{n} r_t$ is the average yield per an $n$-day period. Thus, the realized volatility calculated as the standard deviation of return on equity (close-close price estimator — CC), characterizes the spread of possible returns of equity about the mean value of return.

Another type of historic volatility is Parkinson volatility [4], which employs the maximum and minimum prices of financial asset during a day (high and low prices). Finding the extreme prices requires continuous monitoring; on the other hand, extreme values carry more information than open or close prices do. Volatility returns to the mean value once extreme values are achieved. The value of the indicator based on maximum/minimum prices (high-low price estimator — HL) allows tracking volatility extrema and predicting its further variation. The realized volatility can be easimated as

$$\sigma_{p} = k \sqrt{\frac{1}{n} \sum_{t=1}^{n} p_t^2},$$

(5)

where $p_t = \ln \frac{H_t}{L_t}$, $L_t$ and $H_t$ are the low and high prices of financial asset, respectively, at day $t$, $t=1:n$, and

$k = \frac{1}{2 \sqrt{\ln 2}} \approx 0.601$. 

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