ON MEASURING AND PROFILING CATASTROPHIC RISKS

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An approach to decision-making under catastrophic risks based on risk profiling is proposed. The approach assumes, for some selected catastrophic scenarios, to simulate their consequences (damage) as functions of control parameters and to impose expert constraints on acceptable levels of relative losses in such scenarios. The approach is illustrated by a number of one-stage decision-making problems reduced to mixed linear-programming problems.

Keywords: catastrophic risk, catastrophic flood, risk profiling, risk measures, risk functions, indifference curves, decision-making, investments, risk zoning, insurance.

INTRODUCTION

The paper discusses decision-making under catastrophic risks. By catastrophic risk is meant the probability of a large but improbable loss. There are many publications on decision-making under risk conditions. The following approaches are widely used: minimax approach, expected utility optimization, minimization of mean damage or probability of an unfavorable event, two- and multistage models of stochastic programming, etc. However, there is still no clear or conventional understanding in the theoretical literature how to make a decision under catastrophic risks. In this connection, we propose to account for catastrophic risks by adding catastrophic loss constraints to the constraints of the standard decision-making problem. Namely, for selected catastrophic events (scenarios), we propose to model their consequences (damage) as functions of controlling parameters and to impose expert constraints (define an acceptable level of relative damage) on each catastrophic scenario, i.e., to profile catastrophic risks. Military operations, where possible losses are estimated or planned for each scenario, are elaborated in a similar way. In such a formulation, the probability of each catastrophic scenario does not appear explicitly in the problem formulation, and implicitly corresponds to the level of admissible losses in the constraint for the given scenario. It is assumed here that the less the probability of the catastrophic scenario, the greater the admissible losses. This approach is illustrated with the problem of making investment decisions in catastrophic flood-hazard regions.

1. PROBLEM OF DECISION-MAKING UNDER CATASTROPHIC RISKS

This section discusses approaches to decision-making under catastrophic risks. The theory of stochastic programming, which claims for a methodology of decision-making under risk and uncertainty conditions, offers certain approaches to the formalization of decision-making under such conditions. However, these formulations are not enough for decision-making under catastrophic risks. Such risks imply probability of highly improbable but very large losses, which question the survival of the system. It must be a challenge to estimate either the low probabilities of catastrophic events or large damage caused by them. Clearly, traditional risk indices such as variance are

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inadequate in this case. Mean efficiency indices are also of no use since they are justified in the case of multiply repeating situations, while catastrophes are unique, they might happen tomorrow or never.

Decision-making under risk of natural catastrophes (flood, drought, hurricane, earthquake, etc.), man-caused accidents, and ecological catastrophes is discussed in [1–6], [7, 8], and [9], respectively. Decision-making under catastrophic risks is formalized in [2, 3] as a stochastic programming problem with probabilistic constraints, where the mean efficiency index of a decision is optimized when the catastrophic loss probability is higher than some level. By introducing risk penalties, it is possible to reduce this problem to a standard stochastic programming problem (without probabilistic constraints), risks being minimized in average though large risks being fined with a larger weight.

The problem of decision-making under catastrophic man-caused risks is conceptually formulated in [7, 8] as a problem of two-stage stochastic programming, where decisions of the first stage are made before a catastrophe and decisions of the second stage involve minimization of the damage caused by the catastrophe. The first-stage decisions influence both damage and probabilities of accident scenarios; therefore, the general problem is nonlinear with respect to the variables that describe the decisions.

In [9], an ecological catastrophe is related to loss of stability of a nonlinear deterministic dynamic ecological and economical system, and the distance to the critical points in the phase space where the instability occurs is taken as a measure of risk. Factors of uncertainty and stochasticity in the behavior of the system are not taken into account explicitly. The problem of decision-making is conceptually formulated in terms of cost and benefit, taking a distance to the critical points as a constraint.

It is possible to control catastrophic risks, for example, by estimating and reducing the probability of catastrophic events, which is common practice in the theory of reliability (failure) of complex systems. If a failure due to a catastrophic event is not complete, then it is possible to simulate incomplete failure or damage and to try to minimize it [10]. To this end, portfolio investment theory uses a damage quantile, i.e., a damage level (value at risk (VaR)) such that the probability that it will be exceeded is no greater than a given (small) value. Another index is an average over \( \alpha \) worst damages (conditional value at risk (\( \alpha \)-CVaR)). However, there are usually many unfavorable scenarios, and it is necessary to control the risks associated with all of these scenarios. In this case, risk profiling is used. To profile risks, Rockafellar and Uryasev [11] used a set of conditional average \( \alpha \)-CVaR with different \( \alpha \). This approach is appeal since it allows us to reduce the risk control problem to a (large-dimensional) linear programming problem. However, risk profiling in [11] is indirect because the conditional averages over subsets of scenarios are controlled rather than the risk of each catastrophic scenario. Pursuing the same objective, Kirilyuk [12] proposed to use polyhedral risk measures (including CVaR), which also reduce the decision-making problem to a linear programming one. Control of risk profile is also discussed in [13]. In insurance, catastrophic risks (ruin) are allowed for by introducing an explicit constraint on ruin probability [14]. In [2, 3], a stochastic programming problem with probabilistic constraints is reduced to a two-stage unconstrained stochastic programming problem. Computing a ruin probability is a challenge. Major efforts are made in insurance mathematics to compute or estimate this probability [14–18]. Therefore, it is proposed in [19, 20] to use the Cramer–Lundberg estimate rather than its exact value, i.e., to limit not the probability but its estimate.

The present paper offers another approach to decision-making under catastrophic risks: optimize the average efficiency of a decision under normal (non-catastrophic) conditions with constraints on relative loss due to catastrophic scenarios. When there are many scenarios, they all should and have to be allowed for by aggregating or averaging over scenarios. However, catastrophic events or scenarios should be treated separately. Let us consider a situation where decisions cannot change the probability of a catastrophic event (for example, the probability of a natural catastrophe). In such cases, our proposition is to supplement the constraints of the standard decision-making problem with a set of catastrophic loss constraints, i.e., to specify a profile of catastrophic risks. In this case, the probability of catastrophic scenario does not participate explicitly in the problem formulation, and implicitly corresponds with the level of admissible losses in the constraint for this scenario. It is assumed that heavy losses are admissible for catastrophic scenarios with smaller probability.

**Example** (catastrophic floods). Floods are one of the most destructive natural phenomena. They inevitably happen from time to time on all rivers and natural drainage systems. Flood damages are increased by deforestation, destruction of natural drainage systems, expansion of economic activities into high-water beds of rivers, and global warming. These factors increase the frequency, affected area, rate, and destructive force of floods. As other natural catastrophes, floods are beyond human authority. They become a problem since human activity spread to flood lands. Floods mean potential losses, and it is necessary to take all measures to decrease the losses they cause. The most popular flood alleviation measures are to develop reliable flood prediction systems, construct dams and reservoirs, and to strengthen beds. There are also nonstructural legislative and economic measures such as zoning of economic and building activities, insurance of losses, differentiation of