MATHEMATICAL MODELING OF THE DYNAMICS OF CONSOLIDATION PROCESSES WITH RELAXATION EFFECTS

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The consolidation of salt-saturated porous media is modeled with space–time nonlocality effects and with/without the relaxation of the filtration rate taken into account. A numerical algorithm for modeling the dynamics of the process is proposed and an asymptotic analysis of the problem for excess head is performed for weak spatial nonlocality.

Keywords: mathematical modeling, consolidation, mass transfer, relaxation, nonclassical models, systems of partial differential equations, boundary-value problems, asymptotic approximations, numerical solutions.

INTRODUCTION

Since the anthropogenic impact on the environment intensifies, theoretical study of man-made environmental factors has become especially important. One of the adverse effects of human activity is the pollution of soil and subsoil waters by various chemical substances and industrial wastes. Theoretical studies in this field are of current interest because of not only the practical need for analyzing the conditions of ecologically safe operation of various engineering objects but also the expensive (or sometimes impossible) full-scale modeling of the dynamic processes in these objects. This intensified efforts in mathematical modeling of mass transfer of salt solutions during filtration consolidation of soil foundations of sewage ponds and other hydraulic-engineering objects [1–6]. Note that though methods of mathematical modeling within the framework of this subject have been substantially developed, they are still far from being perfect. For example, the mathematical models of consolidation currently in use disregard some factors that exercise a significant influence on the dynamics of the process under complex mining and geological conditions and in case of a complex microstructure of the salt solutions in sewage ponds. In this connection, it becomes important to take into account the relaxation properties of both the filtration process and deformation of a porous medium [1]. For example, the filtration consolidation of a porous medium saturated with a salt solution is modeled in [2] as a process in a double-relaxation system: relaxation filtration of a pore fluid in a relaxation-compressible medium.

The present paper deals with the mathematical modeling of the filtration consolidation of deformable porous media saturated with salt solutions, with allowance for space–time nonlocality effects [7], which allows, in some cases, more accurate modeling of the dynamics of these processes under complex conditions.

CONSTRUCTING A MATHEMATICAL MODEL REGARDLESS OF FILTRATION RATE RELAXATION. BOUNDARY-VALUE PROBLEM FORMULATION

Let us use the following generalization of Darcy’s law [8, 9] to the motion of salt solutions strongly affected by space–time nonlocality

\[
    u_x = -k \frac{\partial}{\partial x} \left( H + \lambda_1 \frac{\partial H}{\partial t} - \lambda_2 \frac{\partial^2 H}{\partial x^2} \right) \pm v \frac{\partial C}{\partial x},
\]

(1)

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where $H$ is the excess head, $C$ is the salt concentration in the liquid phase, $u_x$ is the filtration rate, $k$ is the filtration coefficient \cite{9}, $\nu$ is the osmotic coefficient (the signs $+$ and $-$ correspond to normal and abnormal osmotic filtrations), $\lambda_i$ ($i = 1, 2$) are relaxation parameters. Note that if $\lambda_1 = \lambda_2 = 0$, relationship (1) yields Darcy’s law generalized to the motion of salt solutions and proposed in \cite{4}. For $\nu = 0$, relationship (1) is the generalized diffusion law considered in \cite{7}.

We get the equation for the excess head from the continuity equation with allowance for the linear law of consolidation \cite{8} by excluding the filtration rate according to (1). As a result,

$$\frac{\partial H}{\partial t} = C\frac{\partial^2 H}{\partial x^2} \left( H + \lambda_1 \frac{\partial H}{\partial t} - \lambda_2 \frac{\partial^2 H}{\partial x^2} \right) + \mu \frac{\partial^2 C}{\partial x^2},$$

(2)

where $C_v$ is a consolidation coefficient \cite{8}, $\mu = \nu C_v k^{-1}$.

Incorporating the filtration rate $u_x$ defined by (1) into the convective diffusion (hydrodynamic dispersion \cite{9}) equation, we obtain

$$\sigma \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} + k \frac{\partial}{\partial x} \left( H + \lambda_1 \frac{\partial H}{\partial t} - \lambda_2 \frac{\partial^2 H}{\partial x^2} \right) \frac{\partial C}{\partial x} + \nu \left( \frac{\partial C}{\partial x} \right)^2,$$

(3)

to determine the salt concentration in the liquid phase; $\sigma$ is the porosity and $D$ is the convective-diffusion coefficient \cite{9}.

The differential equations (2), (3) model filtration consolidation under space–time nonlocality. The model reduces studying the filtration consolidation of a porous soil mass of finite thickness $l$, located, for example, on an impermeable bed to a nonlinear boundary-value problem for system (2), (3) in the domain $(0, l) \times (0, +\infty)$ under the following conditions:

$$H(0, t) = 0, \quad H_x(0, t) = 0,$$

(4)

$$H_x(l, t) = 0, \quad H_{xx}(l, t) = 0,$$

(5)

$$H(x, 0) = H_0,$$

(6)

$$C(0, t) = C_0, \quad C_x(l, t) = 0,$$

(7)

$$C(x, 0) = 0,$$

(8)

where $H_0$ and $C_0$ are the initial excess head and salt concentration at the input of the filtration flow, respectively, which are known.

**APPROXIMATE ALGORITHM TO SOLVE THE BOUNDARY-VALUE PROBLEM**

Let us use the following relationships to introduce dimensionless variables and parameters:

$$x' = \frac{x}{l}, \quad t' = \frac{t}{T}, \quad H' = \frac{H}{H_0}, \quad C' = \frac{C}{C_0}, \quad C_v' = \frac{C_v T}{l^2},$$

$$\nu' = \frac{\nu T C_0}{l^2}, \quad D' = \frac{D T}{l^2}, \quad \xi' = \frac{k H_0 T}{l^2},$$

$$\mu' = \frac{\mu C_0 T}{l^2 H_0}, \quad \lambda'_1 = \frac{\lambda_1}{T}, \quad \lambda'_2 = \frac{\lambda_2}{T}, \quad (D, T, C_0, H_0 = \text{const})$$

(9)

(in what follows, we will omit the prime over dimensionless quantities).

First, let us consider the problem (2), (4)–(6) for the excess head and apply the differential–difference method combined with the method of total representations \cite{10, 11}. To this end, let us introduce a mesh domain $x_i = ih$ ($i = 0, m + 1$) and associate the boundary-value problem with the differential–difference problem (in dimensionless variables (9))