Dynamics of two satellites in the 2/1 Mean–Motion resonance: application to the case of Enceladus and Dione

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Abstract The dynamics of a pair of satellites similar to Enceladus–Dione is investigated with a two-degrees-of-freedom model written in the domain of the planar general three-body problem. Using surfaces of section and spectral analysis methods, we study the phase space of the system in terms of several parameters, including the most recent data. A detailed study of the main possible regimes of motion is presented, and in particular we show that, besides the two separated resonances, the phase space is replete of secondary resonances.

Keywords Enceladus · Dione · Mean–Motion resonance · Periodic orbits · Regular and chaotic motion · Secondary resonance · Saturn

1 Introduction

The ratio of the mean motions of Enceladus and Dione around Saturn is approximately 1.993613, which is very near the ratio 2/1. Due to this commensurability, Enceladus–Dione system (hereafter denoted by E–D) is trapped in a libration such that their conjunctions occur on a line oscillating, around the pericenter of Enceladus, with period 11.5 years and small amplitude of the order of 10.4 (Sinclair 1983). As the pericenters of the orbits of Enceladus and Dione circulate (mainly due to the high perturbation of Saturn oblateness), the previous conjunctions occur at any point of Dione orbit, circulating with period of about 3.8 years.

The role of the 2/1 commensurability in the dynamics of the pair E–D was investigated by several authors using analytical methods (Salgado and Sessin 1985; Bevilacqua and Sessin 1987; Bevilacqua et al. 1989; Message 1999), while others (Ferraz-Mello and Dvorak 1987; Karch and Dvorak 1988; Shinkin 2001) used numerical methods.

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In this work we revisit the same problem, taking a semi-analytical method, where the whole dynamics is studied in the framework of the general three-body planar problem. We have included the oblateness of the planet through $J_2$ and $J_4$ terms, the main critical angles associated to the 2/1 resonance up to power two in the eccentricities (i.e., the two critical angles associated to the resonances of Enceladus and Dione and their first harmonics), and the secular and long-period terms of the disturbing function. Tittemore and Wisdom (1988, 1990) have shown that a similar model has two degrees of freedom, and therefore its dynamics can be studied with the technique of surfaces of section. Here we also use this technique, and the results are checked and complemented with the spectral dynamical map method developed by (Michtchenko and Ferraz-Mello 2001). The aim of this work is to obtain a detailed study of the phase space of the 2/1 Mean–Motion resonance in E–D system.

The current configuration of the E–D pair was probably reached by tidal evolution (Goldreich 1965; Sinclair 1972, 1983; Henrard and Lemaître 1983; Peale 1986, 1999, 2003). It is also believed that the observed resurfacing of Enceladus crust might be related to the resonant dynamics of the system (Peale 2003; Spencer et al. 2006). In this work we will limit ourselves to the conservative dynamics.

We have organized this paper in the following way: the model, some steps in its derivation, and the set of the parameters used are discussed in Sect. 2. In Sect. 3, we show several sets of initial conditions of the pair E–D in different parametric configurations. In Sect. 4, the core of this paper, the phase space of the system in a representative plane of initial conditions, is studied in high detail through surfaces of section and spectral analyses. The conclusions are given in Sect. 5.

2 Model and initial parameters

Let us begin writing the Hamiltonian of a system formed by $N$ satellites with masses $m_i$, orbiting a parent planet with mass $M$. A canonical set of variables introduced by Poincaré (Hori 1985; Laskar and Robutel 1995; Ferraz-Mello et al. 2006) allows us to write the Hamiltonian for the general form of $N + 1$ body problem. Let $\mathbf{r}_i$ be the position vectors of the satellites relative to the center of the planet, and $\mathbf{p}_i$ be the momentum vectors relative to the center of mass of the system. The pair ($\mathbf{r}_i, \mathbf{p}_i$) form a canonical set of variables with the Hamiltonian given by

$$H = H_0 + H_1 + H_J$$

where:

$$H_0 = \sum_{i=1}^{N} \left( \frac{1}{2} \frac{|\mathbf{p}_i|^2}{\beta_i} - \frac{\mu_i \beta_i}{|\mathbf{r}_i|} \right) ,$$

$$H_1 = -G \sum_{0<i<j} \left( \frac{m_i m_j}{\Delta_{ij}} + \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{M} \right) , \quad j = 1, \ldots, N ,$$

$$H_J = -\sum_{i=1}^{N} \frac{\mu_i \beta_i}{|\mathbf{r}_i|} \left[ -\sum_{l=2}^{\infty} J_{l} \left( \frac{R_e}{|\mathbf{r}_i|} \right)^l P_l(\sin \varphi_i) \right] ,$$